S. P.

A

TREATISE

OF

PRACTICAL GEOMETRY.

IN THREE PARTS.

By the late Dr DAVID GREGORY,

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[*Translated from the* Latin. *With Additions.*] By JAMES CRAUFU

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PREFACE.

THIS Treatiſe was compoſed in *Latin* about ſixty Years ago by Dr DAVID GREGORY, then Profeſſor of Mathematicks in the Univerſity of *Edinburgh*; where it has been conſtantly taught ſince that Time, immediately after *Euclid's* Elements and the plain Trigonometry, as proper for exerciſing the Students in the Application of Geometry to Practice. The

Bookſeller

Bookſeller having procured an *Engliſh* Tranſlation of it, which had been made by an ingenious Gentleman when a Student here, this Tranſlation has been reviſed; and ſeveral Additions have been made to the Treatiſe itſelf, in order to render it more uſeful at this Time. The Reader will find theſe diſtinguiſhed from the Author's Text.

COL. M^c^LAURIN.

College of EDINB.
May 1. 1745.

A TREATISE OF *Practical Geometry.*

HAVING explained the first six books of *Euclid*, with the eleventh and twelfth, which may serve for geometrical elements; and having also taught the plain Trigonometry; we are now to subjoin some corollaries which are easily deduced from them, that contain practical rules of great use in the affairs of life, concerning the mensuration of lines, angles, surfaces, and solids.

This Treatise of Practical Geometry is divided into three parts. In the first, we treat of the mensuration of lines and angles; to which we have subjoined Surveying. In the second, we treat of surfaces; not of such as are plain only, but of some curve surfaces

likewiſe; as of the ſurface of the cylinder, cone, and ſphere; and of thoſe parts of the ſphere which we have frequently occaſion to conſider. It is ſhewn how to expreſs the area of theſe in the ſuperficial meaſures that are now in uſe amongſt us. The third part treats of ſolid figures and their menſuration. After deducing the rules for finding the ſolid content of the parallelopipedon, priſm, pyramid, cylinder, cone, &c. from *Euclid*; we add, from *Archimedes*, the menſuration of the ſphere and ſpheroid, and of their ſegments, demonſtrated in an eaſy manner; from whence a method is derived for finding the contents of veſſels that are either full, or in part empty, in the wet, as well as the dry meaſures, that are now in uſe amongſt us.

PART I.

A Line, or length, to be meaſured, whether it be diſtance, height, or depth, is meaſured by a line leſs than it. With us the leaſt meaſure of length is an inch: not that we meaſure no line leſs than it, but becauſe we do not uſe the name of any meaſure

below

below that of an inch; expressing lesser measures by the fractions of an inch: and in this treatise we use decimal fractions as the easiest. Twelve inches make a foot; three feet and an inch make the *Scots* ell; six ells make a fall; forty falls make a furlong; eight furlongs make a mile: So that the *Scots* mile is 1184 paces, accounting every pace to be five feet. These things are according to the statutes of *Scotland*; notwithstanding which, the glaziers use a foot of only eight inches; and other artists for the most part use the *English* foot, on account of the several scales marked on the *English* foot-measure for their use. But the *English* foot is somewhat less than the *Scots*; so that 185 of these, make 186 of those.

Lines, to the extremities and any intermediate point of which you have easy access, are measured by applying to them the common measure a number of times. But lines, to which you cannot have such access, are measured by methods taken from Geometry. The chief whereof we shall here endeavour to explain. The first is by the help of the Geometrical square.

" As for the *English* measures, the yard is " three feet, or thirty-six inches. A pole is " sixteen feet and a half, or five yards and a " half. The chain, commonly called *Gunter's* " *chain*, is four poles, or twenty two yards, " that is sixty six feet. An *English* statute " mile is fourscore chains, or 1760 yards, " that is, 5280 feet.

" The chain (which is now much in use, " because it is very convenient for surveying) " is divided into a hundred links, each of " which is $7\frac{92}{100}$ of an inch: whence it is " easy to reduce any number of those links " to feet, or any number of feet to links.

" A chain that may have the same advan- " tages in surveying in *Scotland*, as *Gunter*'s " chain has in *England*, ought to be in length " seventy four feet, or twenty four *Scots* ells, " if no regard is had to the difference of the " *Scots* and *English* foot above mentioned. " But, if regard is had to that difference, the " *Scots* chain ought to consist of $74\frac{2}{5}$ *English* " feet, or 74 feet 4 inches and $\frac{4}{5}$ of an inch. " This chain being divided into an hundred " links, each of those links is 8 inches and " $\frac{928}{1000}$ of an inch. In the following table,

" the

" the moſt noted meaſures are expreſſed in
" *Engliſh* inches and decimals of an inch."

	Engliſh Inch.	*Dec.*
The *Engliſh* foot, is - -	12	000
The *Paris* foot, - -	12	788
The *Rhinland* foot, meaſured by Mr *Picart*, - -	12	362
The *Scots* foot, - -	12	065
The *Amſterdam* foot, by *Snellius* and *Picart*, - -	11	172
The *Dantzick* foot, by *Hevelius*,	11	297
The *Daniſh* foot, by Mr *Picart*,	12	465
The *Swediſh* foot, by the ſame,	11	692
The *Bruſſels* foot, by Mr *Picart*,	10	828
The *Lyons* foot, by Mr *Auzout*,	13	458
The *Bononian* foot, by Mr *Caſſini*,	14	938
The *Milan* foot, by Mr *Auzout*,	15	631
The *Roman* palm uſed by merchants, according to the ſame, -	9	791
The *Roman* palm uſed by architects,	8	779
The palm of *Naples*, according to Mr *Auzout*, - -	10	314
The *Engliſh* yard, - -	36	000
The *Engliſh* ell, - - -	45	000
The *Scots* ell, - - -	37	200

The

	Inch.	Dec.
The *Paris* aune used by mercers, according to Mr *Picart*,	46	786
The *Paris* aune used by drapers, according to the same,	46	680
The *Lyons* aune, by Mr *Auzout*,	46	570
The *Geneva* aune, - -	44	760
The *Amsterdam* ell, - -	26	800
The *Danish* ell, by Mr *Picart*,	24	930
The *Swedish* ell, - -	23	380
The *Norway* ell, - -	24	510
The *Brabant* or *Antwerp* ell, -	27	170
The *Brussels* ell, - -	27	260
The *Bruges* ell, - - -	27	550
The brace of *Bononia*, according to *Auzout*, - -	25	200
The brace used by architects in *Rome*,	30	730
The brace used in *Rome* by merchants,	34	270
The *Florence* brace used by merchants, according to *Picart*,	22	910
The *Florence* geographical brace,	21	570
The vara of *Seville*, - - -	33	127
The vara of *Madrid*, - -	39	166
The vara of *Portugal*, - -	44	031
The cavedo of *Portugal*, -	27	354
The ancient *Roman* foot, -	11	632

The

	Inch.	Dec.
The *Persian* arish, according to Mr *Greaves*, - -	38	364
The shorter pike of *Constantinople*, according to the same,	25	576
Another pike of *Constantinople*, according to Mess. *Mallet* and *De la Porte*, - - -	27	920

PROPOSITION I.

PROBLEM I.

To describe the structure of the Geometrical square.

THE Geometrical square is made of any solid matter, as brass or wood, or of any four plain rulers joined together at right angles, (as in Fig. 1.); where A is the centre, from which hangs a thread with a small weight at the end, so as to be directed always to the centre. Each of the sides BE and DE is divided into an hundred equal parts, or (if the sides be long enough to admit of it) into a thousand parts; C and F are two sights, fixed on the side AD. There is moreover an index

index GH, which, when there is occaſion, is joined to the centre A, in ſuch manner as that it can move round, and remain in any given ſituation. On this index are two ſights perpendicular to the right line going from the centre of the inſtrument; theſe are K and L. The ſide DE of the inſtrument is called the upright ſide; BE the reclining ſide.

PROP. II. FIG. 2.

To meaſure an acceſſible height, AB, *by the help of a Geometrical ſquare, its diſtance being known.*

LET BR be an horizontal plan, on which there ſtands perpendicularly any line AB: Let BD, the given diſtance of the obſervator from the height, be 96 feet; let the height of the obſervator's eye be ſuppoſed 6 feet; and let the inſtrument held by a ſteady hand, or rather leaning on a ſupport, be directed towards the ſummit A, ſo that one eye (the other being ſhut) may ſee it clearly through the ſights; the perpendicular or plum-line mean while hanging free, and touching the ſurface of the inſtrument: Let now the perpendicular

pendicular be ſuppoſed to cut off on the right ſide KN 80 equal parts. It is clear that LKN, ACK, are ſimilar triangles; for the angles LKN, ACK, are right angles, and therefore equal: moreover LN and AC are parallel, as being both perpendicular to the horizon; conſequently, by *Prop.* 29. 1. *B.* of *Euclid*, the angles KLN, KAC, are equal, wherefore, by the ſecond corollary and of the 32. *Prop.* 1. *B.* of *Euclid*, the angles LNK, and AKC, are likewiſe equal: So that in the triangles NKL, KAC, (by the 4. *Prop.* of the 6. *B.* of *Euclid*) as NK : KL : : KC (*i. e.* BD) : CA; that is, as 80 to 100, ſo is 96 feet to CA. Therefore, by the rule of three, CA will be found to be 120 feet; and CB, which is 6 feet, being added, the whole height is 126 feet.

But if the diſtance of the obſervator from the height, as BE, be ſuch, that, when the inſtrument is directed as formerly toward the ſummit A, the perpendicular falls on the angle P, oppoſite to H, the centre of the inſtrument, and BE or CG be given of 120 feet; CA will alſo be 120 feet. For in the triangles HGP, ACG, aequiangular,

as in the preceeding caſe, as PG : GH : : GC : CA. But PG is equal to GH; therefore GC is likewiſe equal to CA : that is, CA will be 120 feet, and the whole height 126 feet as before.

Let the diſtance BF be 300 feet, and the perpendicular or plum-line cut off 40 equal parts from the reclining ſide : Now, in this caſe, the angles QAC, QZI, are equal, by the 29. *Prop.* 1. *B.* of *Euclid.* And, by the ſame *Prop.* the angles QZI, ZIS, are equal; therefore the angle ZIS, is equal to the angle QAC. But the angles ZSI, QCA are equal, being right angles; therefore in the aequiangular triangles ACQ, SZI, by the 4. *Prop.* of the 6 *B.* of *Euclid,* it will be, as ZS : SI : : CQ : CA; that is, as 100 to 40, ſo is 300 to CA. Wherefore, by the rule of three, CA will be found to be of 120 feet. And, by adding the height of the obſervator, the whole BA will be 126 feet. Note, That the height is greater than the diſtance, when the perpendicular cuts the right ſide, and leſs, if it cut the reclined ſide; and that the height and diſtance are equal, if the perpendicular fall on the oppoſite angle.

SCHOLIUM.

SCHOLIUM. Fig. 3.

If the height of a tower to be measured as above, end in a point, as in Fig. 3. the diſtance of the obſervator oppoſite to it, is not CD, but is to be accounted from the perpendicular to the point A; that is, to CD muſt be added the half of the thickneſs of the tower, *viz.* BD: Which muſt likewiſe be underſtood in the following Propoſitions, when the caſe is ſimilar.

PROP. III.

Prob. Fig. 4.

From the height of a tower AB *given, to find a diſtance on the horizontal plane* BC, *by the Geometrical ſquare.*

LET the inſtrument be ſo placed, as that the mark C in the oppoſite plane may be ſeen through the ſights; and let it be obſerved how many parts are cut off by the perpendicular. Now, by what hath been already demonſtrated, the triangles AEF, ABC, are ſimilar; therefore, by 4th, 6. *Eucl.* it

it will be as EF, to AE ſo AB (compoſed of the height of the tower BG, and of the height of the centre of the inſtrument A, above the tower BG) to the diſtance BC. Wherefore, if, by the rule of three, you ſay, as EF to AE, ſo is AB to BC, it will be the diſtance ſought.

PROP. IV. FIG. 5.

To meaſure any diſtance at land or ſea, by the Geometrical ſquare.

IN this operation, the index is to be applied to the inſtrument, as was ſhown in the deſcription; and, by the help of a ſupport, the inſtrument is to be placed horizontally at the point A; then let it be turned till the remote point F, whoſe diſtance is to be meaſured, be ſeen through the fixed ſights; and bringing the index to be parallel with the other ſide of the inſtrument, obſerve by the ſights upon it any acceſſible mark B, at a ſenſible diſtance: then carrying the inſtrument to the point B, let the immoveable ſights be directed to the firſt ſtation A, and the ſights of the index to the point F.

If

f the index cut the right ſide of the ſquare, s in K, in the two triangles BRK, and BAF, which are æquiangular, it will be (by 4th 6. *Eucl.*) as BR to RK, ſo BA (the diſtance of the ſtations to be meaſured with a chain) to AF; and the diſtance AF ſought will be found by the rule of three. But if the index cut the reclined ſide of the ſquare in any point L, where the diſtance of a more remote point is ſought; in the triangles BLS, BAG, the ſide LS ſhall be to SB, as BA to AG, the diſtance ſought; which accordingly will be found by the rule of three.

PROP. V.

PROB. FIG. 6.

To meaſure an acceſſible height by means of a plain Mirror.

LET AB be the height to be meaſured; let the Mirror be placed at C, in the horizontal plane BD, at a known diſtance BC; let the obſerver go back to D, till he ſee the image of the ſummit in the Mirror, at a certain point of it, which he muſt diligently mark; and let DE be the height of the obſervator's eye. The triangles ABC and EDC are

are æquiangular; for the angles at D and B are right angles; and ACB, ECD, are equal, being the angles of incidence and reflexion of the ray AC, as is demonstrated in optics; wherefore the remaining angles at A, and E, are also equal: Therefore, by 4th, 6. *Eucl.* it will be, as CD to DE, so CB to BA; that is, as the distance of the observator from the point of the Mirror in the right line betwixt the observator and the height, is to the height of the observator's eye, so is the distance of the tower from that point of the Mirror, to the height of the tower sought; which therefore will be found by the rule of three.

Note 1. The observator will be more exact, if, at the point D, a staff be placed in the ground perpendicularly, over the top of which the observator may see a point of the glass exactly in a line betwixt him and the tower.

Note 2. In place of a Mirror may be used the surface of water contained in a vessel, which naturally becomes parallel to the horizon.

PROP.

PROP. VI. FIG. 7.

To measure an accessible height AB *by means of two staffs.*

LET there be placed perpendicularly in the ground a longer staff DE, likewise a shorter one FG, so as the observator may see A, the top of the height to be measured, over the ends D, F, of the two staffs; let FH and DC, parallel to the horizon, meet DE and AB in H and C; then the triangles FHD, DCA, shall be æquiangular; for the angles at C and H are right ones; likewise the angle A, is equal to the angle FDH, by 29. 1. *Eucl.*; wherefore the remaining angles DFH, and ADC, are also equal: wherefore, by 4. 6. *Eucl.* as FH, the distance of the staffs, to HD, the excess of the longer staff above the shorter; so is DC, the distance of the longer staff from the tower, to CA, the excess of the height of the tower above the longer staff. And thence CA will be found by the rule of three.

To which if the length DE be added, you will have the whole height of the tower BA. Q. E. F.

SCHO-

SCHOLIUM. Fig. 8.

Many other methods may be occasionally contrived for measuring an accessible height. For example, from the given length of the shadow BD, I find out the height AB, thus: Let there be erected a staff CE perpendicularly, producing the shadow EF: The triangles ABD, CEF, are æquiangular; for the angles at B, and E, are right; and the angles ADB, and CFE, are equal, each being equal to the angle of the sun's elevation above the horizon: Therefore, by 4th, 6. *Eucl.* as EF the shadow of the staff, to EC the staff itself, so BD the shadow of the tower, to BA the height of the tower. Tho' the plane on which the shadow of the tower falls be not parallel to the horizon, if the staff be erected in the same plane, the rule will be the same.

PROP. VII.

To measure an inaccessible height by means of two staffs.

HITHERTO we have supposed the height to be accessible, or that we can come at the lower end of it; now if, because of

ſome impediment, we cannot get to a tower, or if the point whoſe height is to be found out, be the ſummit of a hill, ſo that the perpendicular be hid within the hill; if, I ſay, for want of better inſtruments, ſuch an inacceſſible height is to be meaſured by means of two ſtaffs, let the firſt obſervation be made with the ſtaffs DE and FG, as in *Prop.* I.; then the obſervator is to go off in a direct line from the height and firſt ſtation, till he come to the ſecond ſtation; where he is to place the longer ſtaff perpendicularly at RN, and the ſhorter ſtaff at KO, ſo that the ſummit A may be ſeen along their tops; that is, ſo that the points KNA may be in the ſame right line. Through the point N let there be drawn the right line NP parallel to FA: Whereſore in the triangles KNP, KAF, the angles KNP, KAF are equal, by the 29. 1. *Eucl.* alſo the angle AKF is common to both; conſequently the remaining angle KPN, is equal to the remaining angle KFA. And therefore, by 4th, 6. *Eucl.* PN : FA : : KP : KF. But the triangles PNL, FAS are ſimilar; therefore, by 4th, 6. *Eucl.*

PN : FA : : NL : SA. Therefore, by the 11. 5. *Eucl.* KP : KF : : NL ; SA. Thence, alternately, it will be, as KP (the exceſs of the greater diſtance of the ſhort ſtaff from the long one above its leſſer diſtance from it) to NL, the exceſs of the longer ſtaff above the ſhorter; ſo KF, the diſtance of the two ſtations of the ſhorter ſtaff, to SA, the exceſs of the height ſought above the height of the ſhorter ſtaff. Wherefore SA will be found by the rule of three. To which let the height of the ſhorter ſtaff be added, and the ſum will give the whole inacceſſible height BA. Q. E. F.

Note 1. In the ſame manner may an inacceſſible height be found by a Geometrical ſquare, or by a plain Speculum. But we ſhall leave the rules to be found out by the ſtudent, for his own exerciſe.

Note 2. That by the height of the ſtaff we underſtand its height above the ground in which it is fixed.

Note 3. Hence depends the method of uſing other inſtruments invented by Geometricians; for example, of the Geometrical croſs: And if all things be juſtly weighed,

a like rule will serve for it as here. But we incline to touch only upon what is most material.

PROP. VIII. FIG. 9.

To measure the distance AB, *to one of whose extremities we have access, by the help of four staffs.*

LET there be a staff fixed at the point A; then going back at some sensible distance in the same right line, let another be fixed in C, so as that both the points A and B be covered and hid by the staff C; likewise going off in a perpendicular from the right line CB, at the point A, (the method of doing which shall be shown in the following *Scholium*), let there be placed another staff at H; and in the right line CKG (perpendicular to the same CB, at the point C), and at the point of it K, such that the points K, H, and B, may be in the same right line, let there be fixed a fourth staff. Let there be drawn, or let there be supposed to be drawn, a right line HG parallel to CA. The triangles KHG, HAB will be æquiangular; for the angles HAB, KGH are right angles. Also, by

29th, 1. *Eucl.* the angles ABH, KHG are equal; wherefore, by the 4th, 6. *Eucl.* as KG (the excess of CK above AH) to GH, or to CA, the distance betwixt the first and second staff; so is AH, the distance betwixt the first and third staff, to AB the distance sought.

SCHOLIUM. Fig. 10.

To draw on a plane a right line AE perpendicular to CH, from a given point A; take the right lines AB, AD, on each side equal; and in the points B and D, let there be fixed stakes, to which let there be tied two equal ropes BE, DE, (or one having a mark in the middle, and holding in your hand their extremities joined, (or the mark in the middle, if it be but one), draw out the ropes on the ground; and then, where the two ropes meet, or at the mark, when by it the rope is fully stretched, let there be placed a third stake at E; the right line AE will be perpendicular to CH in the point A, by 11th, 1. *Eucl.* In a manner not unlike to this may any problems that are resolved by the square and compasses, be done by ropes and a cord turned round as a radius.

PROP.

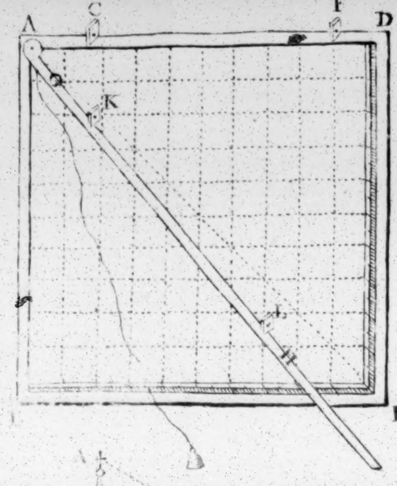


FIG. 1.

FIG. 2.

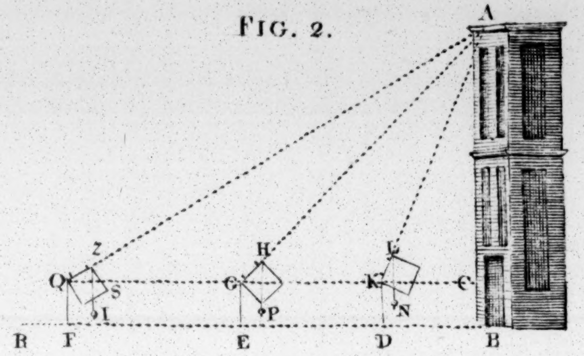


FIG. 3.

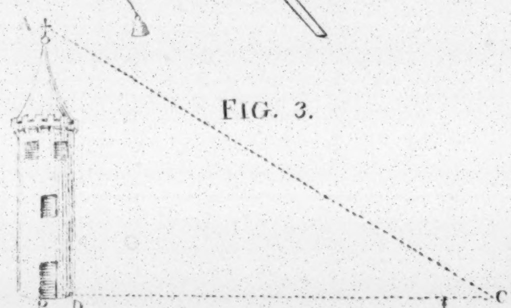
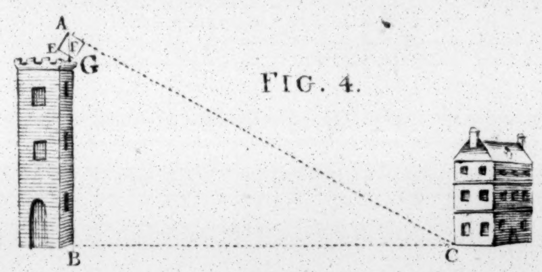


FIG. 4.



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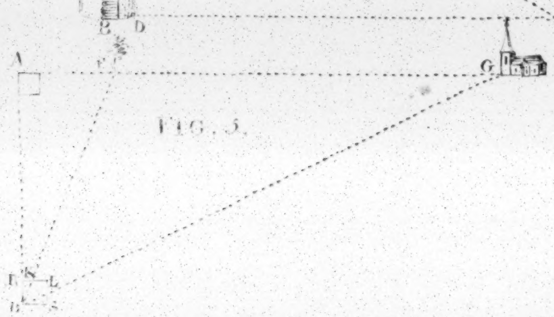

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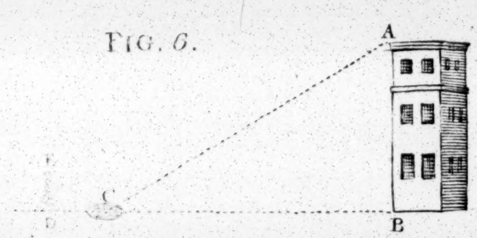


FIG. 7.

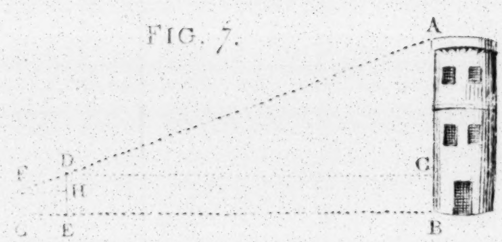


FIG. 8.

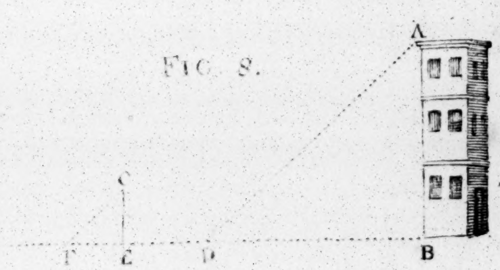


FIG. 10

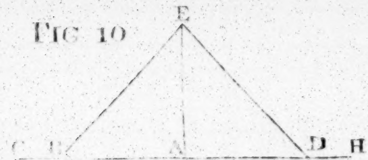


FIG. 9.

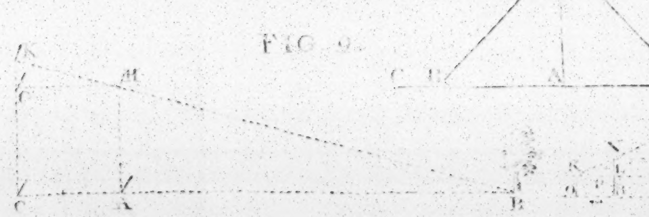
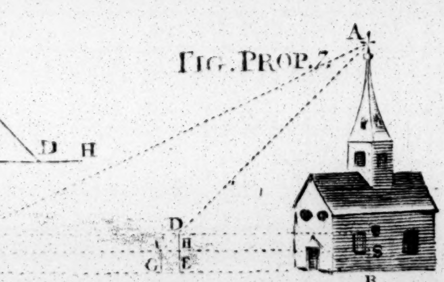


FIG. PROP. 7.



PROP. IX. FIG. 11.

To measure the distance AB, *one of whose extremities is accessible.*

FROM the point, let the right line AC of a known length be made perpendicular to AB, (by the preceeding *Scholium*) : likewise draw the right line CD perpendicular to CB, meeting the right line AB in D: then, by the 8. 6. *Eucl.* as DA : AC : : AC : AB. Wherefore when DA and AC are given, AB will be found by the rule of three. Q. E. F.

SCHOLIUM.

All the preceeding operations depend on the equality of some angles of triangles, and on the similarity of the triangles arising from that equality. And on the same principles depend innumerable other operations which a Geometrician will find out of himself, as is very obvious. However some of these operations require such exactness in the work, and without it are so liable to errors, that, *caeteris paribus*, the following operations which are performed by a trigonometrical calculation,

calculation, are to be preferred; yet could we not omit those above, being most easy in practice, and most clear and evident to those who have only the first elements of Geometry. But if you are provided with instruments, the following operations are more to be relied upon. We do not insist on the easiest cases to those who are skilled in plain trigonometry, which is indeed necessary to any one who would apply himself to practice. It would be easy to the reader to find examples; and we have shown in plain trigonometry how to find the angle or side of any plain triangle that is required, from the angles or sides that may be given.

PROP. X. FIG. 12.

To describe the construction and use of the Geometrical Quadrant.

THE Geometrical Quadrant is the fourth part of a circle divided into ninety degrees, to which two sights are adapted, with a perpendicular or plum-line hanging from the centre. The general use of it is for investigating angles in a vertical plane, compre-

hended

hended under right lines going from the centre of the instrument, one of which is horizontal, and the other is directed to some visible point. This instrument is made of any solid matter, as wood, copper, *&c.*

PROP. XI. Fig. 13.

To describe the make and use of the Graphometer.

THE Graphometer is a semicircle made of any hard matter, of wood, for example, or brass, divided into 180 degrees; so fixed on a *fulcrum*, by means of a brass ball and socket, that it easily turns about, and retains any situation; two sights are fixed on its diameter. At the centre there is commonly a magnetical needle in a box. There is likewise a moveable ruler, which turns round the centre, and retains any situation given it. The use of it is to observe any angle, whose vertex is at the centre of the instrument in any plane, (though it is most commonly horizontal, or nearly so), and to find how many degrees it contains.

PROP. XII.

FIG. 14. and 15.

To describe the manner in which angles are measured by a Quadrant or Graphometer.

LET there be an angle in a vertical plane, comprehended between a line parallel to the horizon HK, and the right line RA, coming from any remarkable point of a tower or hill, or from the sun, moon, or a star. Suppose that this angle RAH is to be measured by the Quadrant: let the instrument be placed in the vertical plane, so as that the centre A may be in the angular point; and let the sights be directed towards the object at R, (by the help of the ray coming from it, if it be the sun or moon, or by the help of the visual ray, if it is any thing else), the degrees and minutes in the arch BC cut off by the perpendicular, will measure the angle RAH required. For, from the make of the Quadrant, BAD is a right, angle; therefore BAR is likewise right being equal to it. But, because HK is horizontal, and AC perpendicular, HAC will be

be a right angle; and therefore equal alſo to BAR. From thoſe angles ſubtract the part HAB that is common to both; and there will remain the angle BAC equal to the angle RAH. But the arch BC is the meaſure of the angle BAC; conſequently, it is likewiſe the meaſure of the angle RAH.

Note, that the remaining arch on the Quadrant DC is the meaſure of the angle RAZ, comprehended between the foreſaid right line RA and AZ which points to the Zenith.

Let it now be required to meaſure the angle ACB (Fig. 15.) in any plane, comprehended between the right lines AC and BC, drawn from two points A and B, to the place of ſtation C. Let the Graphometer be placed at C, ſupported by its *fulcrum* (as was ſhown above); and let the immoveable ſights on the ſide of the inſtrument DE be directed towards the point A; and likewiſe (while the inſtrument remains immoveable) let the ſights of the ruler FG (which is moveable about the centre C) be directed to the point B. It is evident that the moveable ruler cuts off an arch DH, which is the meaſure of the angle ACB ſought. Moreover, by the ſame

method, the inclination of DE, or of FG, may be obſerved with the meridian line, which is pointed out by the magnetick needle incloſed in the box, and is moveable about the centre of the inſtrument, and the meaſure of this inclination or angle found in degrees.

PROP. XIII. Fig. 16.

To meaſure an acceſſible height by the Geometrical Quadrant.

BY the 12th *Prop.* of this part, let the angle C be found by means of the Quadrant. Then in the triangle ABC, right-angled at B, (BC being ſuppoſed the horizontal diſtance of the obſervator from the tower), having the angle at C, and the ſide BC, the required height BA will be found by the 3d caſe of plain Trigonometry.

PROP. XIV. Fig. 17.

To meaſure an inacceſſible height by the Geometrical Quadrant.

LET the angle ACB be obſerved with the Quadrant (by the 12th *Prop.* of this part); then let the obſerver go from C to the ſecond

ſecond ſtation D, in the right line BCD (providing BCD be a horizontal plane); and after meaſuring this diſtance CD, take the angle ADC likewiſe with the Quadrant. Then, in the triangle ACD, there is given the angle ADC, with the angle ACD; becauſe ACB was given before: therefore (by 32. 1. *Eucl.*) the remaining angle CAD is given likewiſe. But the ſide CD is likewiſe given, being the diſtance of the ſtation C and D; therefore (by the firſt caſe of oblique-angled triangles in Trigonometry) the ſide AC will be found. Wherefore in the right-angled triangle ABC, all the angles and the hypothenuſe AC are given; conſequently, by the 4th caſe of Trigonometry, the height ſought AB will be found; as alſo (if you pleaſe) the diſtance of the ſtation C from AB the perpendicular within the hill or inacceſſible height.

PROP. XV. FIG. 18.

From the top of a given height, to meaſure the diſtance BC.

LET the angle BAC be obſerved by the 12th of this part; wherefore in the triangle ABC, right-angled at B, there is given

method, the inclination of DE, or of FG, may be observed with the meridian line, which is pointed out by the magnetick needle inclosed in the box, and is moveable about the centre of the instrument, and the measure of this inclination or angle found in degrees.

PROP. XIII. FIG. 16.

To measure an accessible height by the Geometrical Quadrant.

BY the 12th *Prop.* of this part, let the angle C be found by means of the Quadrant. Then in the triangle ABC, right-angled at B, (BC being supposed the horizontal distance of the observator from the tower), having the angle at C, and the side BC, the required height BA will be found by the 3d case of plain Trigonometry.

PROP. XIV. FIG. 17.

To measure an inaccessible height by the Geometrical Quadrant.

LET the angle ACB be observed with the Quadrant (by the 12th *Prop.* of this part); then let the observer go from C to the second

second station D, in the right line BCD (providing BCD be a horizontal plane); and after measuring this distance CD, take the angle ADC likewise with the Quadrant. Then, in the triangle ACD, there is given the angle ADC, with the angle ACD; because ACB was given before: therefore (by 32. 1. *Eucl.*) the remaining angle CAD is given likewise. But the side CD is likewise given, being the distance of the station C and D; therefore (by the first case of oblique-angled triangles in Trigonometry) the side AC will be found. Wherefore in the right-angled triangle ABC, all the angles and the hypothenuse AC are given; consequently, by the 4th case of Trigonometry, the height sought AB will be found; as also (if you please) the distance of the station C from AB the perpendicular within the hill or inaccessible height.

PROP. XV. FIG. 18.

From the top of a given height, to measure the distance BC.

LET the angle BAC be observed by the 12th of this part; wherefore in the triangle ABC, right-angled at B, there is given

given by obſervation the angle at A; whence (by the 32. 1. *Eucl.)* there will alſo be given the angle BCA: moreover the ſide AB (being the height of the tower) is ſuppoſed to be given. Wherefore, by the 3d caſe of Trigonometry, BC the diſtance ſought will be found.

PROP. XVI. FIG. 19.

To meaſure the diſtance of two places A *and* B, *of which one is acceſſible, by the Graphometer.*

LET there be erected at two points A and C, ſufficiently diſtant, two viſible ſigns; then (by the 12th of this) let the two angles BAC, BCA be taken by the Graphometer. Let the diſtance of the ſtations A and C be meaſured with a chain. Then the third angle B being known, and the ſide AC being likewiſe known; therefore, by the firſt caſe of Trigonometry, the diſtance required, AB, will be found.

PROP.

PROP. XVII. FIG. 20.

To measure by the Graphometer, the distance of two places, neither of which is accessible.

LET two stations C and D be chosen, from each of which the places may be seen whose distance is sought: let the angles ACD, ACB, BCD, and likewise the angles BDC, BDA, CDA, be measured by the Graphometer; let the distance of the stations C and D be measured by a chain, or (if it be necessary) by the preceeding practice. Now in the triangle ACD, there are given two angles ACD and ADC; therefore the third CAD is likewise given: Moreover the side CD is given; therefore, by the first case of Trigonometry, the side AD will be found. After the same manner, in the triangle BCD, from all the angles and one side CD given, the side BD is found. Wherefore in the triangle ADB, from the given sides DA and DB, and the angle ADB contained by them, the side AB (the distance sought) is found by the 4th case of Trigonometry of oblique-angled triangles.

Let it be noted, that it is not necessary that

that the points A, B, C, and D, be in one plane; and that any triangle is in one plane by 2d *Prop.* 11th of *Eucl.*

PROP. XVIII. FIG. 21.

It is required by the Graphometer and Quadrant, to measure an accessible height AB, *placed so on a steep, that one can neither go near it in an horizontal plane, nor recede from it, as we supposed in the solution of the* 14*th* Prop.

LET there be chosen any situation as C, and another D; where let some mark be erected: let the angles ACD and ADC be found by the Graphometer; then the third angle DAC will be known. Let the side CD, the distance of the stations, be measured with a chain, and thence (by Trigon.) the side AC will be found. Again, in the triangle ACB, right-angled at B, having found by the Quadrant the angle ACB, the other angle CAB is known likewise: but the side AC in the triangle ADC is already known; therefore the height required AB will be found by the 4th case of right-angled triangles.

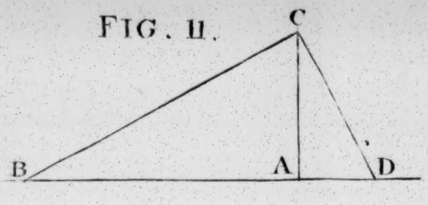


FIG. 11.

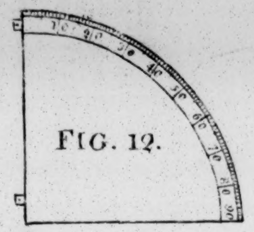


FIG. 12.

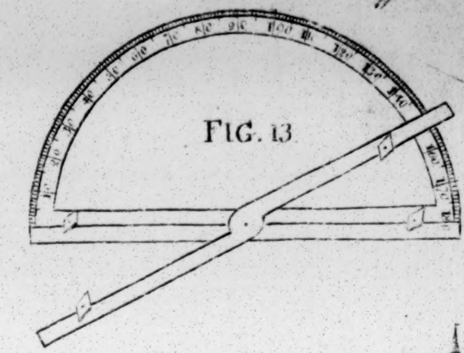


FIG. 13

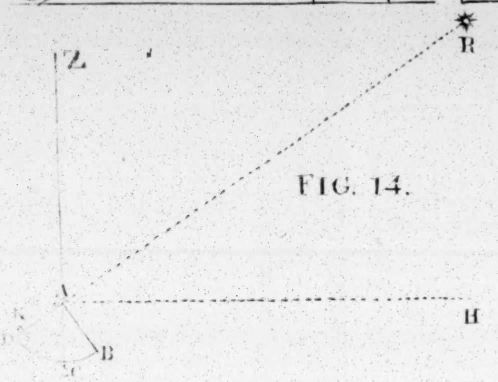


FIG. 14.

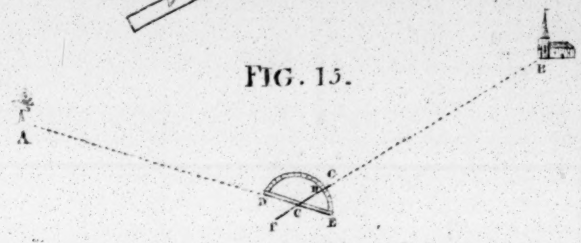


FIG. 15.

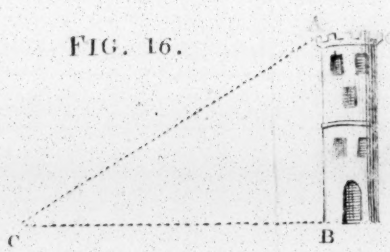


FIG. 16.

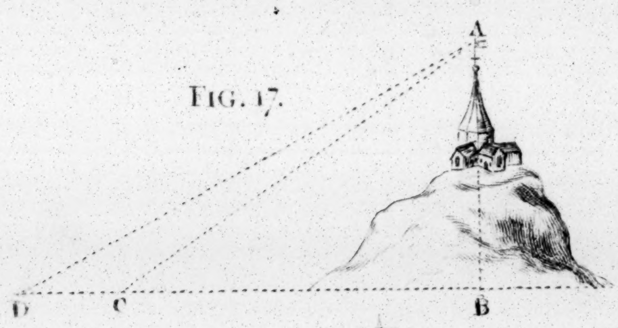


FIG. 17.

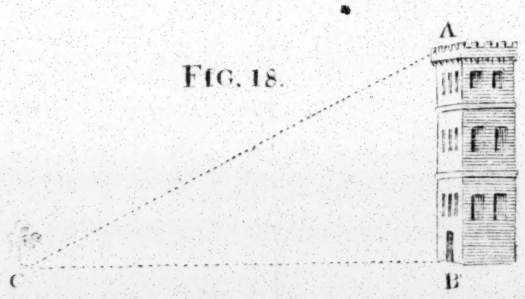


FIG. 18.

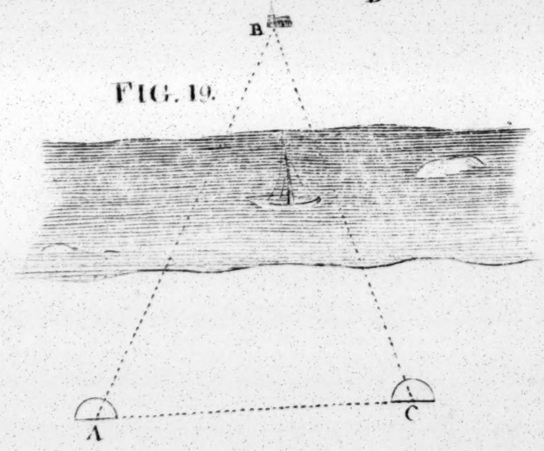


FIG. 19.

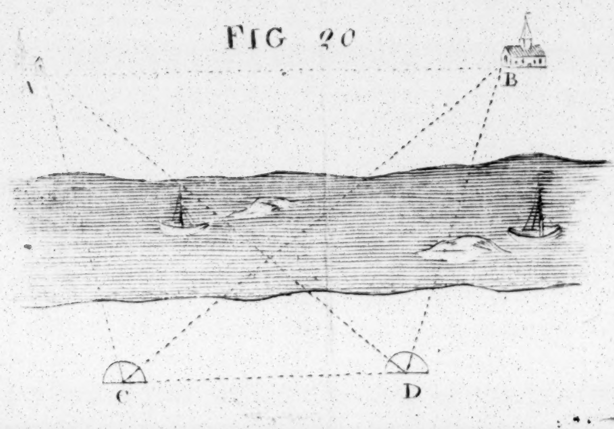


FIG. 20

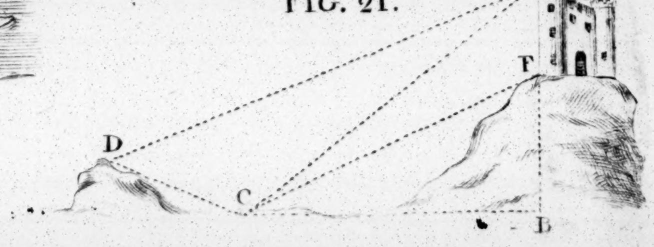


FIG. 21.

angles. If the height of the tower is wanted, the angle BCF will be found by the Quadrant; which being taken from the angle ACB already known, the angle ACF will remain: but the angle FAC was known before; therefore the remaining angle AFC will be known. But the ſide AC was alſo known before; therefore, in the triangle AFC, all the angles and one of the ſides AC being known, AF the height of the tower above the hill will be found by Trigonometry.

SCHOLIUM.

It were eaſy to add many other methods of meaſuring heights and diſtances: but, if what is above be underſtood, it will be eaſy (eſpecially for one that is verſed in the elements) to contrive methods for this purpoſe, according to the occaſion: ſo that there is no need of adding any more of this ſort. We ſhall ſubjoin here a method by which the diameter of the earth may be found out.

PROP.

PROP. XIX. FIG. 22.

To find the diameter of the earth from one observation.

LET there be chosen a high hill AB, near the sea-shore, and let the observator on the top of it, with an exact Quadrant divided into minutes and seconds by transverse divisions, and fitted with a telescope in place of the common sights, measure the angle ABE contained under the right line AB, which goes to the centre, and the right line BE drawn to the sea, a tangent to the globe at E; let there be drawn from A perpendicular to BD, the line AF meeting BE in F. Now in the right-angled triangle BAF all the angles are given, also the side AB, the height of the hill; which is to be found by some of the foregoing methods, as exactly as possible; and, by Trigonometry, the sides BF and AF are found. But, by Corol. 36. 3d *Eucl.* AF is equal to FE; therefore BE will be known. Moreover, by 36th, 3d *Eucl.* the rectangle under BA and BD, is equal to the square of BE. And thence, by 17th, 6. *Eucl.* as AB : BE : : BE : BD. Therefore,

fore, ſince AB and BE are already given, BD will be found by 11th, 6. *Eucl.* or by the rule of three; and ſubſtracting BA, there will remain AD the diameter of the earth ſought.

SCHOLIUM.

Many other methods might be propoſed for meaſuring the diameter of the earth. The moſt exact in my opinion is that propoſed by Mr *Picart*, of the academy of ſciences at *Paris*. But ſince it does not belong to this place, we refer you to the philoſophical tranſactions, where you will find it deſcribed.

" According to Mr *Picart*, a degree of " the meridian at thel atitude of 49° 21′ was " 57,060 *French Toiſes*, each of which con- " tains ſix feet of the ſame meaſure; from " which it follows, that if the earth be " an exact ſphere, the circumference of a " great circle of it will be 123,249,600 " *Paris* feet, and the ſemidiameter of the " earth 19,615,800 feet: but the *French* " Mathematicians, who of late have exami- " ned Mr *Picart*'s operations, aſſure us, " That the degree in that latitude is 57, 183 " *Toiſes*. They meaſured a degree in *Lap-*

" *land*, in the latitude of 66° 20′, and found " it of 57,438 *Toises*. By comparing these " degrees, as well as by the observations " on pendulums, and the theory of gravity, " it appears that the earth is an oblate sphe- " roid; and (supposing those degrees to be " accurately measured) the axis or diameter " that passes through the poles will be to " the diameter of the equator, as 177 " to 178, or the earth will be 22 miles " higher at the equator than at the poles. " A degree has likewise been measured at " the equator, and found to be considera- " bly less than at the latitude of *Paris*; " which confirms the oblate figure of the " earth. But an account of this last men- " suration has not been published as yet. " If the earth was of an uniform density " from the surface to the centre, then, ac- " cording to the theory of gravity, the me- " ridian would be an exact ellipsis, and the " axis would be to the diameter of the equa- " tor as 230 to 231; and the difference of " the semidiameter of the equator and semi- " axis about 17 miles."

In what follows, a figure is often to be

laid down on paper, like to another figure given ; and becauſe this likeneſs conſiſts in the equality of their angles, and in the ſides having the ſame proportion to each other (by the definitions of the 6th of *Eucl.*) we are now to ſhow what methods practical Geometricians uſe for making on paper an angle equal to a given angle, and how they conſtitute the ſides in the ſame proportion. For this purpoſe they make uſe of a Protractor, (or, when it is wanting, a Line of chords), and of a Line of equal parts.

PROP. XX.

FIG. 23, 24, 25, 26, and 27.

To deſcribe the conſtruction and uſe of the Protractor, of the Line of chords, and of the Line of equal parts.

THE Protractor is a ſmall ſemicircle of braſs, or ſuch ſolid matter. The ſemicircumference is divided into 180 degrees. The uſe of it is, to draw angles on any plane, as on paper, or to examine the extent of angles already laid down. For this laſt purpoſe, let the ſmall point in the centre

of

of the Protractor be placed above the angular point, and let the ſide AB coincide with one of the ſides that contain the angle propoſed; the number of degrees cut off by the other ſide, computing on the Protractor from B, will ſhow the quantity of the angle that is to be meaſured.

But if an angle is to be made of a given quantity on a given line, and at a given point of that line, let AB coincide with the given line, and let the centre A of the inſtrument be applied to that point. Then let there be a mark made at the given number of degrees; and a right line drawn from that mark to the given point, will conſtitute an angle with the given right line, of the quantity required; as is manifeſt.

This is the moſt natural and eaſy method, either for examining the extent of an angle on paper, or for deſcribing on paper an angle of a given quantity.

But when there is ſcarcity of inſtruments, or becauſe a line of chords is more eaſily carried about, (being deſcribed on a ruler on which there are many other lines beſides.) practical Geometricians frequently

make

make use of it. It is made thus: let the Quadrant of a circle be divided into 90 degrees; (as in Fig. 24.). The right line AB is the chord of 90 deg.; the chord of every arch of the Quadrant is transferred to this line AB, which is always marked with the number of degrees in the corresponding arch.

the sines

of

Note, That the chord of 60 degrees is equal to the radius, by Corol. 15. 4th *Eucl.* If now a given angle EDF is to be measured by the Line of chords, from the centre D, with the distance DG, (the chord of 60 degrees), describe the arch GF; and let the points G and F be marked where this arch intersects the sides of the angle. Then if their distance GF, applied on the line of chords from A to B, gives (for example) 25 degrees, this shall be the measure of the angle proposed.

When an obtuse angle is to be measured with this line, let its complement to a semicircle be measured, and thence it will be known. It were easy to transfer to the diameter of a circle the chords of all arches to the extent of a semicircle; but such are rarely found marked upon rules.

But

But now, if an angle of a given quantity, suppose of 50 degrees, is to be made at a given point M of the right line KL (Fig. 26.) From the centre M, and the distance MN equal to the chord of 60 degrees, describe the arch QN. Take off an arch NR, whose chord is equal to that of 50 degrees on the Line of chords; join the points M and R; and it is plain that MR shall contain an angle of 50 degrees with the line KL proposed.

But sometimes we cannot produce the sides, till they be of the length of a chord of 60 degrees on our scale; in which case it is fit to work by a circle of proportions (that is a sector), by which an arch may be made of a given number of degrees to any radius.

The quantities of angles are likewise determined by other lines usually marked upon rules, as the lines of sines, tangents, and secants; but as these methods are not so easy or so proper in this place, we omit them.

To delineate figures similar or like to others given, besides the equality of the angles, the same proportion is to be preserved among the sides of the figures that is

to

to be delineated, as is among the ſides of the figure given. For which purpoſe, on the rules uſed by artiſts, there is a line divided into equal parts; more or leſs in number, and greater or leſſer in quantity, according to the pleaſure of the maker.

A foot is divided into inches; and an inch, by means of tranſverſe lines, into 100 equal parts; ſo that with this ſcale, any number of inches, below twelve, with any part of an inch, can be taken by the compaſſes, providing ſuch part be greater than the one hundredth part of an inch. And this exactneſs is very neceſſary in delineating the plans of houſes, and in other caſes.

PROP. XXI. FIG. 28.

To lay down on paper, by the Protractor or Line of chords, and Line of equal parts, a right-lined figure like to one given, providing the angles and ſides of the figure given be known by obſervation or menſuration.

FOR example, ſuppoſe that it is known that in a quadrangular figure, one ſide is of 235 feet, that the angle contained by it and the ſecond ſide is of 84°, the ſecond ſide of

of 288 feet, the angle contained by it and the third ſide of 72°, and that the third ſide is 294 feet. Theſe things being given, a figure is to be drawn on paper like to this quadrangular figure. On your paper, at a proper point A, let a right line be drawn, upon which take 235 equal parts, as AB. The part repreſenting a foot is taken greater or leſſer, according as you would have your figure greater or leſs. In the adjoining figure, the 100th part of an inch is taken for a foot. And accordingly an inch divided into an 100 parts, and annexed to the figure, is called a ſcale of 100 feet. Let there be made at the point B (by the preceeding *Prop.*) an angle ABC of 84°, and let BC be taken of 288 parts like to the former. Then let the angle BCD be made of 72°, and the ſide CD of 294 equal parts. Then let the ſide AD be drawn; and it will compleat the figure like to the figure given. The meaſures of the angle A and D can be known by the Protractor or Line of chords, and the ſide AD by the Line of equal parts; which will exactly anſwer to the correſponding angles and to the ſide of the primary figure.

After

After the very ſame manner, from the ſides and angles given, which bound any right-lined figure, a figure like to it may be drawn, and the reſt of its ſides and angles be known.

COROLLARY.

Hence any trigonometrical problem in right-lined triangles, may be reſolved by delineating the triangle from what is given concerning it, as in this Propoſition. The unknown ſides are examined by a line of equal parts, and the angles by a protractor or line of chords.

PROP. XXII. PROB.

The diameter of a circle being given, to find its circumference nearly.

THE periphery of any polygon inſcribed in the circle is leſs than the circumference, and the periphery of any polygon deſcribed about a circle is greater than the circumference. Whence *Archimedes* firſt diſcovered that the diameter was in proportion to the circumference, as 7 to 22 nearly; which

ſerves for common uſe. But the moderns have computed the proportion of the diameter to the circumference to greater exactneſs. Suppoſing the diameter 100, the periphery will be more than 314, but leſs than 315*. But *Ludolphus van Cuelen* exceeded the labours of all; for by immenſe ſtudy he found, that, ſuppoſing the diameter

100,000,000,000,000,000,000,000,000,000,000,

the periphery will be leſs than

314,159,265,358,979,323,846,264,338,327,951,

but greater than

314,159,265,358,979,323,846,264,338,327,950;

whence it will be eaſy, any part of the circumference being given in degrees and minutes, to aſſign it in parts of the diameter.

Of ſurveying and meaſuring of LAND.

HITHERTO we have treated of the meaſuring of angles and ſides, whence it is abundantly eaſy to lay down a field, a plane, or an entire country: For to this nothing is requiſite but the protraction of triangles, and of other plain figures, after having

* The diameter is more nearly to the circumference, as 113 to 355.

ving meaſured their ſides and angles. But as this is eſteemed an important part of practical Geometry, we ſhall ſubjoin here an account of it, with all poſſible brevity; ſuggeſting withal, that a Surveyor will improve himſelf more by one day's practice, than by a great deal of reading.

PROP. XXIII. Prob.

To explain what Surveying is, and what inſtruments Surveyors uſe.

FIRST, it is neceſſary that the Surveyor view the field that is to be meaſured, and inveſtigate its ſides and angles, by means of an iron chain (having a particular mark at each foot of length, or at any number of feet, as may be moſt convenient for reducing lines or ſurfaces to the received meaſures*), and the Graphometer deſcribed above. *Secondly*, It is neceſſary to delineate the field *in plano*, or to form a map of it; that is, to lay down on paper a figure ſimilar to the field;

* See above p. 4. the account of Gunter's chain, and of the chain that is moſt convenient for meaſuring land in Scotland.

field ; which is done by the Protractor (or line of chords) and of the line of equal parts. *Thirdly*, It is neceſſary to find out the area of the field ſo ſurveyed and repreſented by a map. Of this laſt we are to treat below, in the ſecond part.

The ſides and angles of ſmall fields are ſurveyed by the help of a plain-table; which is generally of an oblong rectangular figure, and ſupported by a *fulcrum*, ſo as to turn every way by means of a ball and ſocket. It has a moveable frame, which ſurrounds the board, and ſerves to keep a clean paper put on the board cloſe and tight to it. The ſides of the frame facing the paper are divided into equal parts every way. The board hath beſides a box with a magnetic needle, and moreover a large index with two ſights. On the edge of the frame of the board are marked degrees and minutes, ſo as to ſupply the room of a Graphometer.

PROP.

PROP. XXIV.

PROB. FIG. 29.

To delineate a field by the help of a plain-table, from one station whence all its angles may be seen, and their distances measured by a chain.

LET the field that is to be laid down be ABCDE. At any convenient place F, let the plain-table be erected; cover it with clean paper, in which let some point near the middle represent the station. Then applying at this place the index with the sights, direct it so as that through the sights some mark may be seen at one of the angles, suppose A; and from the point F, representing the station, draw a faint right line along the side of the index: then, by the help of the chain, let FA the distance of the station from the foresaid angle be measured. Then taking what part you think convenient for a foot or pace from the Line of equal parts, set off on the faint line the parts corresponding to the line FA that was measured; and let there be a mark made representing the angle of the field A. Keeping

ing the table immoveable, the ſame is to be done with the reſt of the angles; then right lines joining thoſe marks ſhall include a figure like to the field, as is evident from 5. 6. *Eucl.*

COROLLARY.

The ſame thing is done in like manner by the Graphometer; for having obſerved in each of the triangles, AFB, BFC, CFD, *&c.* the angle at the ſtation F, and having meaſured the lines from the ſtation to the angles of the field, let ſimilar triangles be protracted on paper (by the 21. of this) having their common vertex in the point of ſtation. All the lines, excepting thoſe which repreſent the ſides of the field, are to be drawn faint or obſcure.

Note 1. When a Surveyor wants to lay down a field, let him place diſtinctly in a regiſter all the obſervations of the angles, and the meaſures of the ſides, until, at time and place convenient, he draw out the figure on paper.

Note 2. The obſervations made by the help of the Graphometer are to be examined; for all the angles about the point F ought to be equal to four right ones by 13th, 1. *Eucl.*

PROP.

PROP. XXV.

PROB. FIG. 30.

To lay down a field by means of two ſtations, from each of which all the angles can be ſeen, by meaſuring only the diſtance of the ſtations.

LET the inſtrument be placed at the ſtation F; and having choſen a point repreſenting it upon the paper which is laid upon the plain-table, let the index be applied at this point, ſo as to be moveable about it. Then let it be directed ſucceſſively to the ſeveral angles of the field; and when any angle is ſeen through the ſights, draw an obſcure line along the ſide of the index. Let the index, with the ſights, be directed after the ſame manner to the ſtation G; on the obſcure line drawn along its ſide, pointing to A, ſet off from the Scale of equal parts a line correſponding to the meaſured diſtance of the ſtations; and this will determine the point G. Then remove the inſtrument to the ſtation G; and applying the index to the line repreſent-

ing

ing the diſtance of the ſtations, place the inſtrument ſo that the firſt ſtation may be ſeen through the ſights. Then the inſtrument remaining immoveable, let the index be applied at the point repreſenting the ſecond ſtation G; and be ſucceſſively directed by means of its ſights, to all the angles of the field, drawing (as before) obſcure lines; and the interſection of the two obſcure lines that were drawn to the ſame angle from the two ſtations will always repreſent that angle on the plan. Care muſt be taken that thoſe lines be not miſtaken for one another. Lines joining thoſe interſections will form a figure on the paper like to the field

SCHOLIUM.

It will not be difficult to do the ſame by the Graphometer, if you keep a diſtinct account of your obſervations of the angles made by the line joining the ſtations, and the lines drawn from the ſtations to the reſpective angles of the field. And this is the moſt common manner of laying down whole countries. The tops of two mountains are taken for two ſtations, and their

their diſtance is either meaſured by ſome of the methods mentioned above, or is taken according to common repute. The ſights are ſucceſſively directed towards cities, churches, villages, forts, lakes, turnings of rivers, woods, *&c.*

Note, The diſtance of the ſtations ought to be great enough, with reſpect to the field that is to be meaſured; ſuch ought to be choſen as are not in a line with any angle of the field. And care ought to be taken likewiſe that the angles, for example, FAG, FDG, *&c.* be neither very acute, nor very obtuſe. Such angles are to be avoided as much as poſſible; and this admonition is found very uſeful in practice.

PROP. XXVI.

PROB. FIG. 31.

To lay down any field, however irregular its figure may be, by the help of the Graphometer.

LET ABCEDHG be ſuch a field. Let its angles (in going round it) be obſerved with a Graphometer (by the 12th of

this) and noted down; let its ſides be meaſured with a chain; and (by what was ſaid on the 21ſt of this) let a figure like to the given field be protracted on paper. If any mountain is in the circumference, the horizontal line hid under it is to be taken for a ſide, which may be found by two or three obſervations according to ſome of the methods deſcribed above; and its place on the map is to be diſtinguiſhed by a ſhade, that it may be known a mountain is there.

If not only the circumference of the field is to be laid down in the plan, but alſo its contents, as villages, gardens, churches, public roads, we muſt proceed in this manner.

Let there be (for example) a church F, to be laid down in the plan. Let the angles ABF, BAF be obſerved and protracted on paper in their proper places, the interſection of the two ſides BF and AF will give the place of the church on the paper: or, more exactly, the lines BF, AF being meaſured, let circles be deſcribed from the centers B and A, with parts from the ſcale correſponding to the diſtances BF and AF, and the place of the church will be at their interſection.

Note

Note 1. While the angles obſerved by the Graphometer are taken down, you muſt be careful to diſtinguiſh the external angles, as E and G, that they may be rightly protracted afterwards on paper.

Note 2. Our obſervations of the angles may be examined by computing if all the internal angles make twice as many right angles, four excepted, as there are ſides of the figure: for this is demonſtrated by 32d, 1. *Eucl.* But in place of any external angle DEC, its complement to a circle is to be taken.

PROP. XXVII.

PROB. FIG. 32.

To lay down a plain field without inſtruments.

IF a ſmall field is to be meaſured, and a map of it to be made, and you are not provided with inſtruments; let it be ſuppoſed to be divided into triangles, by right-lines, as in the figure; and after meaſuring the three ſides of any of the triangles, for example of ABC, let its ſides be laid down from a convenient ſcale on paper, by the

the 22d of this. Again, let the other two ſides BD, CD of the triangle CBD be meaſured and protracted on the paper by the ſame ſcale as before. In the ſame manner proceed with the reſt of the triangles of which the field is compoſed, and the map of the field will be perfected; for the three ſides of a triangle determine the triangle; whence each triangle on the paper is ſimilar to its correſpondent triangle in the field, and is ſimilarly ſituated: conſequently the whole figure is like to the whole field.

SCHOLIUM.

If the field be ſmall, and all its angles may be ſeen from one ſtation, it may be very well laid down by the Plain-table by the 24th of this. If the field be larger, and have the requiſite conditions, and great exactneſs is not expected, it likewiſe may be plotted by means of the Plain-table, or by the Graphometer, according to the 25th of this; but in fields that are irregular and mountainous, when an exact map is required, we are to make uſe of the Graphometer, as in the 26th of this, but rarely of the Plain-table.

Having

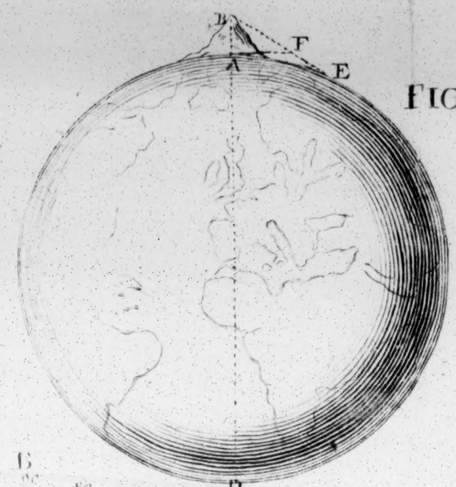
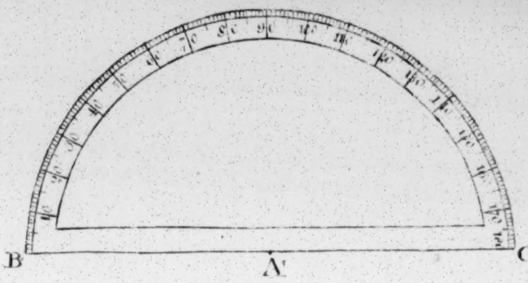


FIG. 22.

FIG. 23.



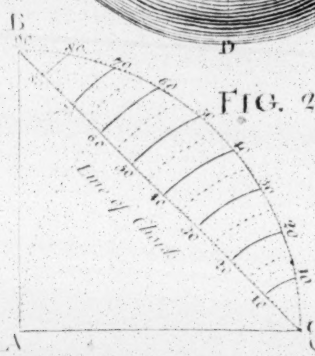


FIG. 24.

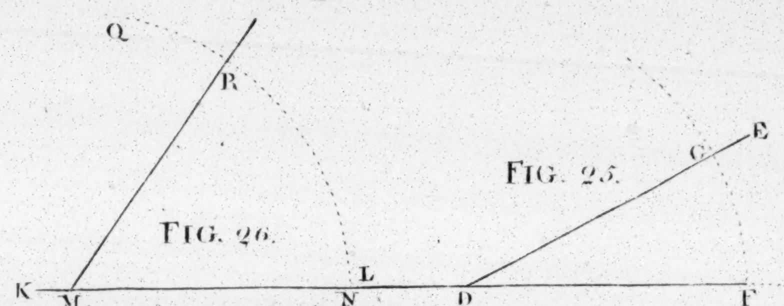


FIG. 26.

FIG. 25.

Scale of half an Inch divided into 100 parts

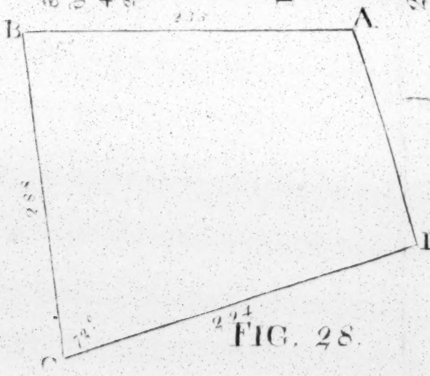


FIG. 28.

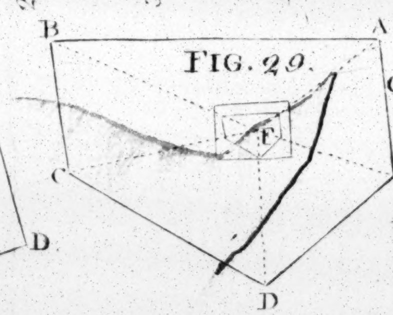


FIG. 29.

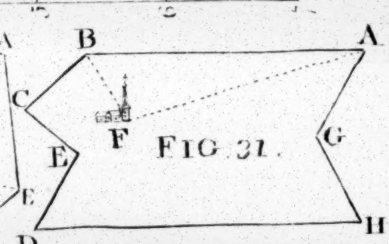


FIG. 31.

FIG. 32.

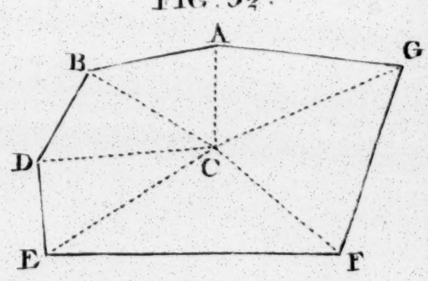
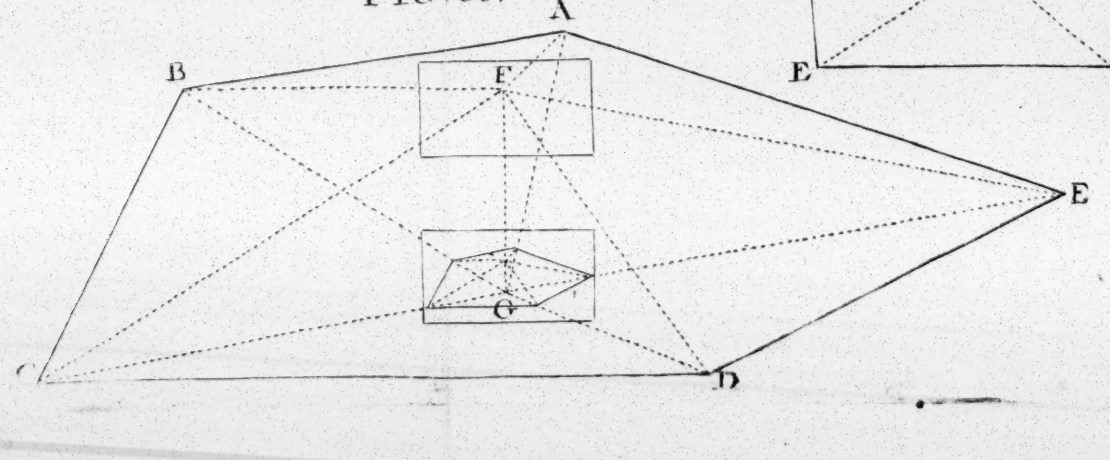


FIG. 30.



Having protracted the bounding lines, the particular parts contained within them may be laid down by the proper operations for this purpose, delivered in the 26th Proposition; and the method described in the 27th Proposition may be sometimes of service; for we may trust more to the measuring of sides, than to the observing of angles. We are not to compute four-sided and many-sided figures till they are resolved into triangles; for the sides do not determine those figures.

In the laying down of cities, or the like, we may make use of any of the methods described above that may be most convenient.

The map being finished, it is transferred on clean paper, by putting the first sketch above it, and marking the angles by the point of a small needle. These points being joined by right lines, and the whole illuminated by colours proper to each part, and the figure of the Mariner's compass being added to distinguish the North and South, with a scale on the margin, the map or plan will be finished and neat.

We have thus briefly and plainly treated of Surveying, and shown by what instruments it is performed; having avoided those methods

methods which depend on the magnetick needle, not only becauſe its direction may vary in different places of a field (the contrary of this at leaſt doth not appear), but becauſe the quantity of an angle obſerved by it cannot be exactly known; for an error of two or three degrees can ſcarcely be avoided in taking angles by it. As for the remaining part of Surveying, whereby the area of a field already laid down on paper is found in acres, roods, or any other ſuperficial meaſures; this we leave to the following part, which treats of the menſuration of ſurfaces.

" Beſides the inſtruments deſcribed above, " a Surveyor ought to be provided with an " Off-ſet ſtaff, equal in length to ten links " of the chain, and divided into ten equal " parts. He ought likewiſe to have ten " arrows or ſmall ſtreight ſticks near two " feet long, ſhod with iron ferrils. When " the chain is firſt opened, it ought to be " examined by the Off-ſet ſtaff. In mea-" ſuring any line, the leader of the chain " is to have the ten arrows at firſt ſetting " out. When the chain is ſtretched in the " line, and the near end touches the place

" from

"from which you meaſure, the leader ſticks "one of the ten arrows in the ground, at "the far end of the chain. Then the leader "leaving the arrow, proceeds with the chain "another length; and the chain being "ſtretched in the line, ſo that the near end "touches the firſt arrow, the leader ſticks "down another arrow at his end of the "chain. The line is preſerved ſtreight, if "the arrows be always ſet ſo as to be in "a right line with the place you meaſure "from, and that to which you are going. "In this manner they proceed till the leader "have no more arrows. At the eleventh "chain, the arrows are to be carried to him "again, and he is to ſtick one of them into "the ground, at the end of the chain. And "the ſame is to be done at the 21, 31, 41, "*&c.* chains, if there are ſo many in the "right line to be meaſured. In this man-"ner you can hardly commit an error in "numbering the chains, unleſs of ten chains "at once.

"The Off-ſet ſtaff ſerves for meaſuring "readily the diſtances of any things pro-"per to be repreſented in your plan, from

"the

" the ſtation-line while you go along. Theſe " diſtances ought to be entred into your " field-book, with the correſponding diſtan- " ces from the laſt ſtation, and proper re- " marks, that you may be enabled to plot " them juſtly, and be in no danger of mi- " ſtaking one for another, when you extend " your plan. The field-book may be con- " veniently divided into five pages. In the " middle-column the angles at the ſeveral " ſtations taken by the Theodolite are to be " entered, with the diſtances from the ſta- " tions. The diſtances taken by the Off-ſet " ſtaff, on either ſide of the ſtation-line, are " to be entered into columns on either ſide " of the middle-column, according to their " poſition with reſpect to that line. The " names or characters of the objects, with " proper remarks, may be entered in columns " on either ſide of theſe laſt.

" Becauſe, in the place of the Grapho- " meter deſcribed by our Author, Survey- " ors now make uſe of the Theodolite, we " ſhall ſubjoin a deſcription of Mr *Siſſon*'s " lateſt improved Theodolite from Mr *Gard-*

" *ner*'s

" *ner's practical Surveying improved.* See a " figure of it, in plate 4.

" In this inſtrument, the three ſtaffs, by " braſs ferrils at top, ſcrew into bell-metal " joints, that are moveable between braſs pil- " lars, fixed in a ſtrong braſs plate ; in which " round the centre is fixed a ſocket with a " ball moveable in it, and upon which the " four ſcrews preſs, that ſet the Limb hori- " zontal : Next above is another ſuch plate, " through which the ſaid ſcrews paſs, and on " which, round the centre, is fixed a fruſtum " of a cone of bell-metal, whoſe axis (being " connected with the centre of the ball is al- " ways perpendicular to the Limb, by means " of a conical braſs ferril fitted to it, whereon " is fixed the Compaſs-box ; and on it the " Limb, which is a ſtrong bell-metal ring, " whereon are moveable three braſs indexes ; " in whoſe plate are fixed four braſs pillars, " that, joining at top, hold the centre-pin of " the bell-metal double Sextant, whoſe double " index is fixed on the centre of the ſame " plate : within the double Sextant is fixed the " ſpirit-level, and over it the Teleſcope.

" The Compaſs-box is graved with two

" diamonds for *North* and *South*, and with
" 20 degrees on both ſides of each, that the
" needle may be ſet to the variation, and its
" error alſo known.

" The Limb has two *Fleur de luces* againſt
" the diamonds in the box, inſtead of 180
" each; and is curiouſly divided into whole
" degrees, and numbered to the left hand
" at every ten to twice 180, having three
" indexes diſtant 120, (with *Nonius*'s diviſions
" on each for the decimals of a degree), that
" are moved by a pinion fixed below one of
" them, without moving the Limb; and in
" another is a ſcrew and ſpring under, to fix
" it to any part of the Limb. It has alſo
" diviſions numbered, for taking the quarter
" girt in inches of round *Timber* at the mid-
" dle height, when ſtanding ten feet horizon-
" tally diſtant from its centre; which at 20
" muſt be doubled, and at 30 trebled; to
" which a ſhorter index is uſed, having *No-*
" *nius*'s diviſions for the decimals of an inch:
" but an abatement muſt be made for the
" bark, if not taken off.

" The double Sextant is divided on one
" ſide from under its centre (when the Spirit-
" tube

" tube and Telescope are level) to above 60 " degrees each way, and numbered at 10, " 20, *&c.* and the double index (through " which it is moveable) shews on the same " side the degree and decimal of any altitude " or depression to that extent by *Nonius*'s di- " visions: on the other side are divisions " numbered, for taking the upright height of " *Timber, &c.* in feet, when distant 10 feet; " which at 20 must be doubled, and at 30 " tripled; and also the quantities for redu- " cing hypothenusal lines to horizontal. It " is moveable by a pinion fixed in the double " index.

" The Telescope is a little shorter than " the diameter of the Limb, that a fall may " not hurt it; yet it will magnify as much, " and shew a distant object as perfect, as " most of treble its length. In its focus are " very fine cross wires, whose intersection is " in the plane of the double Sextant; and " this was a whole circle, and turned in a " lathe to a true plane, and is fixed at right " angles to the Limb; so that, whenever " the Limb is set horizontal, (which is rea- " dily done by making the Spirit-tube level

" over

" over two ſcrews, and the like over the o-
" ther two), the double Sextant and Tele-
" ſcope are moveable in a vertical plane; and
" then every angle taken on the Limb (tho'
" the Teleſcope be never ſo much elevated
" or depreſſed) will be an angle in the plane
" of the horizon. And this is abſolutely
" neceſſary in plotting a horizontal plane.

" If the lands to be plotted are hilly, and
" not in any one plane, the lines meaſured
" cannot be truly laid down on paper, with-
" out being reduced to one plane, which
" muſt be the horizontal, becauſe angles are
" taken in that plane.——

" In viewing my objects, if they have
" much altitude or depreſſion, I either write
" down the degree and decimal ſhewn on the
" double Sextant, or the links ſhewn on the
" back-ſide; which laſt ſubtracted from e-
" very chain in the ſtation-line, leaves the
" length in the horizontal plane. But if the
" degree is taken, the following table will
" ſhew the quantity.

A

A TABLE of the links to be ſubtracted out of every chain in hypothenuſal lines *of ſeveral degrees altitude, or depreſſion, for reducing them to* horizontal.

Degrees.	*Links.*	*Degrees.*	*Links.*	*Degrees.*	*Links,*
4,05	- - $\frac{1}{4}$	14,07	- - 3	23,074	- - 8
5,73	- - $\frac{1}{2}$	16,26	- - 4	24,495	- - 9
7,02	- - $\frac{3}{4}$	18,195	- - 5	25,84	- - 10
8,11	- - 1	19,95	- - 6	27,13	- - 11
11,48	- - 2	21,565	- - 7	28,36	- - 12

" Let the firſt ſtation-line really meaſure " 1107 links, and the angle of altitude or " depreſſion be 19°,93 ; looking in the table " I find againſt 19°,95, is 6 links. Now " 6 times 11 is 66 ; which ſubtracted from " 1107, leaves 1041, the true tength to be " laid down in the plan.

" It is uſeful in ſurveying, to take the an- " gles which the bounding lines form with " the magnetic needle, in order to check " the angles of the figure, and to plot them " conveniently afterwards."

PART

PART II.

Of the surfaces of bodies.

THE smallest superficial measure with us is a square inch; 144 of which make a square foot. Wrights make use of these in the measuring of deals and planks; but the square foot which the Glaziers use in measuring of glass, consists only of 64 square inches. The other measures are, *first*, the ell square; 2*dly*, the fall, containing 36 square ells; 3*dly*, the rood, containing 40 falls, 4*thly*, the acre, containing 4 roods. Slaters, Masons, and Pavers, use the ell square and the fall; Surveyors of land use the square ell, the fall, the rood, and the acre.

The superficial measures of the *English*, are, *first*, the square foot; 2*dly*, the square yard, containing 9 square feet; for their yard contains only 3 feet; 3*dly*, the pole, containing $30\frac{1}{4}$ square yards; 4*thly*, the rood, containing 40 poles; 5*thly*, the acre, containing 4 roods. And hence it is easy to reduce our superficial measures to the *English*, or theirs to ours.

"In

" In order to find the content of a field, it " is most convenient to measure the lines by " the chains described above, *p.* 4 that of " 22 yards for computing the *English* acres, " and that of 24 *Scots* ells for the acres of " *Scotland.* The chain is divided into 100 " links, and the square of the chain is 10,000 " square links; ten squares of the chain, or " 100,000 square links, give an acre. There- " fore if the area be expressed by square links, " divide by 100,000, or cut off five decimal " places, and the quotient shall give the area " in acres and decimals of an acre. Write " the entire acres apart; but multiply the de- " cimals of an acre by 4, and the product " shall give the remainder of the area in roods " and decimals of a rood. Let the entire " roods be noted apart after the acres; then " multiply the decimals of a rood by 40, and " the product shall give rhe remainder of the " area in falls or poles. Let the entire falls or " poles be then writ after the roods, and mul- " tiply the decimals of a fall by 36, if the area " is required in the measures of *Scotland*; but " multiply the decimals of a pole by $30\frac{1}{4}$, if " the area is required in the measures of *Eng-*

" *land,*

" *land*, and the product shall give the remain-
" der of the area in square ells in the former
" case, but in square yards in the latter. If,
" in the former case, you would reduce the
" decimals of the square ell to square feet,
" multiply them by 9.50694; but in the latter
" case, the decimals of the *English* square
" yard are reduced to square feet, by mul-
" tiplying them by 9.

" Suppose, for example, that the area ap-
" pears to contain 12,65842 square links of
" the chain of 24 ells; and that this area is
" to be expressed in acres, roods, falls, *&c.*
" of the measures of *Scotland*. Divide the
" square-links by 100,000, and the quotient
" 12.65842 shows the area to contain 12
" acres $\frac{65842}{100000}$ of an acre. Multiply the
" decimal part by 4, and the product 2.63368
" gives the remainder in roods and decimals
" of a rood. Those decimals of the rood
" being multiplied by 40, the product gives
" 25.3472 falls. Multiply the decimals of
" the fall by 36, and the product gives
" 12.4992 square ells, The decimals of the
" square ell multiplied by 9.50994 give

" 4.748

" 4.7458 ſquare feet. Therefore the area " propoſed amounts to 12 acres, 2 roods, 25 " falls, 12 ſquare ells, and $4\frac{7458}{10000}$ ſquare feet.

" But if the area contains the ſame number " of ſquare links of *Gunter*'s chain, and is to " be expreſſed by *Engliſh* meaſures, the " acres and roods are computed in the ſame " manner as in the former caſe. The poles " are computed as the falls. But the deci- " mals of the pole, *viz.* $\frac{3472}{10000}$ are to be " multiplied by $30\frac{1}{4}$ (or 30. 25), and the pro- " duct gives 10.5028 ſquare yards. The " decimals of the ſquare yard multiplied by " 9, give 4.5252 ſquare feet; therefore in " this caſe the area is in *Engliſh* meaſure 12 " acres, 2 roods, 25 poles, 10 ſquare yards, " and $4\frac{5252}{10000}$ ſquare feet.

" The *Scots* acre is to the *Engliſh* acre, by " ſtatute, as 100,000 to 78,694, if we have re- " gard to the difference betwixt the *Scots* and " *Engliſh* foot above mentioned. But it is cu- " ſtomary in ſome parts of *England* to have " 18, 21, *&c.* feet to a pole, and 160 ſuch " poles to an acre: whereas, by the ſtatute, " $16\frac{1}{2}$ feet make a pole. In ſuch caſes the

" acre is greater in the duplicate ratio of the
" number of feet to a pole.

" They who measure land in *Scotland* by
" an ell of 37 *English* inches, make the acre
" less that the true *Scots* acre by $593\frac{6}{10}$ square
" *English* feet, or by about $\frac{1}{93}$ of the acre.

" An husband-land contains 6 acres of
" sock and sythe land, that is of land that
" may be tilled with a plough, and mown
" with a sythe; 13 acres of arable land make
" an oxgang or oxengate; four oxengate
" make a pound land of old extent (by a de-
" cree of the Exchequer, *March* 11. 1585),
" and is called *librata terræ*. A forty shilling
" land of old extent contains eight oxgang,
" or 104 acres.

" The *arpent* about *Paris* contains 32,400
" square *Paris* feet, and is equal to $2\frac{2}{3}$ *Scots*
" roods or $3\frac{37}{100}$ *English* roods.

" The *actus quadratus*, according to *Varro*
" *Collumella*, &c. was a square of 120 *Roman*
" feet. The *Jugerum* was the double of this.
" 'Tis to the *Scots* acre as 10,000 to 20,456,
" and to the *English* acre as 10,000 to 16,097.
" It was divided (like the *as*) into 12 *unciae*,
" and the *uncia* into 24 *scrupula*." This, with

the

the three preceeding paragraphs, are taken from an ingenious manuscript written by Sir *Robert Stewart* Professor of natural philosophy. The greatest part of the table in *p.* 5. was taken from it likewise.

PROP. I.

PROB. FIG. 1.

To find out the area of a rectangular parallelogram ABCD.

LET the side AB, for example, be five feet long, and BC (which constitutes with BA a right angle at B) be 17 feet. Let 17 be multiplied by 5, and the product 85 will be the number of square feet in the area of the figure ABCD. But if the parallelogram proposed is not rectangular as BEFC, its base BC multiplied into its perpendicular height AB (not into its side BE) will give its area. This is evident from 35th 1. *Eucl.*

PROP. II.

PROB. FIG. 2.

To find the area of a given triangle.

LET the triangle BAC be given, whose base BC is supposed 9 feet long; let the perpen-

perpendicular AD be drawn from the angle A opposite to the base, and let us suppose AD to be four feet. Let the half of the perpendicular be multiplied into the base, or the half of the base into the perpendicular, or take the half of the product of the whole base into the perpendicular, the product gives 18 square feet for the area of the given triangle.

But if only the sides are given, the perpendicular is found either by protracting the triangle, or by 12th and 13th 2. *Eucl.* or by Trigonometry. But how the area of a triangle may be found from the given sides only shall be shewn in the 4th Prop. of this part.

PROP. III.

PROB. FIG. 3.

To find the area of any rectilineal figure.

IF the figure be irregular, let it be resolved into triangles; and drawing perpendiculars to the bases in each of them, let the area of each triangle be found by the preceeding Prop. and the sum of these areas will give the area of the figure.

SCHO-

SCHOLIUM 1.

In meaſuring boards, planks, and glaſs, their ſides are to be meaſured by a foot-rule divided into 100 equal parts; and after multiplying the ſides, the decimal fractions are eaſily reduced to leſſer denominations. The menſuration of theſe is eaſy, when they are rectangular parallelograms.

SCHOLIUM 2.

If a field is to be meaſured, let it firſt be plotted on paper, by ſome of the methods deſcribed in the preceeding part, and let the figure ſo laid down be divided into triangles, as was ſhown in the preceeding Propoſition.

The baſe of any triangle, or the perpendicular upon the baſe, or the diſtance of any two points of the field is meaſured, by applying it to the ſcale according to which the map is drawn.

SCHOLIUM 3.

But if the field given be not in a horizontal plane, but uneven and mountainous, the ſcale gives the horizontal line between any two points, but not their diſtance meaſured on the

uneven

uneven surface of the field. And indeed it would appear that the horizontal plane is to be accounted the area of an uneven and hilly country. For if such ground is laid out for building on, or for planting with trees or bearing corn, since these stand perpendicular to the horizon, it is plain that a mountainous country cannot be considered as of greater extent for those uses than the horizontal plane; nay, perhaps, for nourishing of plants, the horizontal plane may be preferable.

If however the area of a figure as it lies irregularly on the surface of the earth is to be measured, this may be easily done by resolving it into triangles as it lies. The sum of their areas will be the area sought; which exceeds the area of the horizontal figure more or less, according as the field is more or less uneven.

PROP. IV.

PROB. FIG. 2.

The sides of a triangle being given, to find the area, without finding the perpendicular.

LET all the sides of the triangle be collected into one sum; from the half of which let the

the sides be separately subtracted, that three differences may be found betwixt the foresaid half sum and each side; then let these three differences, and the half sum be multiplied into one another, and the square-root of the product will give the area of the triangle. For example, let the sides be 10, 17, 21; the half of their sum is 24; the three differences betwixt this half sum and the three sides, are 14, 7, and 3. The first being multiplied by the second, and their product by the third, we have 294 for the product of the differences; which multiplied by the foresaid half sum 24, gives 7056; the square-root of which 84 is the area of the triangle. The demonstration of this, for the sake of brevity, we omit. It is to be found in several treatises, particularly in *Clavius*'s *Practical Geometry*.

PROP V.

THEOR. FIG. 4.

The area of the ordinate figure ABEFGH *is equal to the product of the half circumference of the polygon multiplied into the perpendicular drawn*

drawn from the centre of the circumscribed circle to the side of the polygon.

FOR the ordinate figure can be resolved into as many equal triangles, as there are sides of the figure; and since each triangle is equal to the product of half the base into the perpendicular, it is evident that the sum of all the triangles together, that is the polygon, is equal to the product of half the sum of the bases (that is the half of the circumference of the polygon) into the common perpendicular height of the triangles drawn from the centre C to one of the sides; for example to AB.

PROP. VI.

PROB. FIG. 5.

The area of a circle is found by multiplying the half of the periphery into the radius, or the half of the radius into the periphery.

FOR a circle is not different from an ordinate or regular polygon of an infinite number of sides, and the common height of the triangles into which the polygon, or circle may

may be ſuppoſed to be divided, is the radius of the circle.

Were it worth while, it were eaſy to demonſtrate accurately this Propoſition by means of the inſcribed and circumſcribed figures, as is done in the 5th Prop. of the treatiſe of *Archimedes* concerning the dimenſions of the circle.

COROLLARY.

Hence alſo it appears that the area of the ſector ABCD is produced, by multiplying the half of the arch into the radius; and likewiſe that the area of the ſegment of the circle ADC is found, by ſubtracting from the area of the ſector the area of the triangle ABC.

PROP. VII.

THEOR. FIG. 6.

The circle is to the ſquare of the diameter, as 11 *to* 14 *nearly.*

FOR if the diameter AB be ſuppoſed to be 7, the circumference AHBK will be almoſt 22 (by the 22d Prop. of the firſt part of

this), and the area of the ſquare DC will be 49; and, by the preceeding Prop. of this, the area of the circle will be $38\frac{1}{2}$: therefore the ſquare DC will be to the inſcribed circle as 49 to $38\frac{1}{2}$, or as 98 to 77, that is, as 14 to 11. *Q. E. D.*

If greater exactneſs is required, you may proceed to any degree of accuracy: for the ſquare DC is to the inſcribed circle, as 1 to $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13}$ *&c. in infinitum.*

" This ſeries will be of no ſervice for com-
" puting the area of the circle accurately,
" without ſome further artifice, becauſe it
" converges at too ſlow a rate. The area of
" the circle will be found exactly enough for
" moſt purpoſes, by multiplying the ſquare
" of the diameter by 7854, and dividing by
" 10,000, or cutting off four decimal places
" from the product; for the area of the cir-
" cle is to the circumſcribed ſquare nearly as
" 7854 to 10,000."

PROP.

PROP. VIII.

PROB. FIG. 7.

To find the area of a given ellipse.

LET ABCD be an ellipse, whose greater diameter is BD and lesser AC, bisecting the greater perpendicularly in E. Let a mean proportional HF be found (by 13th 6. *Eucl.*) between AC and BD, and (by the 6th of this) find the area of the circle described on the diameter HF. I say, that this area is equal to the area of the ellipse ABCD. For because, as BD to AC, so the square of BD to the square of HF, (by 2. *Cor.* 20th 6. *Eucl.*): but (by the 2d 12. *Eucl.*) as the square of BD to the square of HF, so is the circle of the diameter BD to the circle of the diameter HF: therefore as BD to AC, so is the circle of the diameter BD to the circle of the diameter HF. And (by the 5th Prop. of *Archimedes* of spheroids) as the greater diameter BD to the lesser AC, so is the circle of the diameter BD to the ellipse ABCD. Consequently (by the 11th 5. *Eucl.*) the circle of the diameter BD will have the same proportion to the circle of the diame-

ter

ter HF, and to the ellipse ABCD. Therefore, by 9th 5. *Eucl.* the area of the circle of the diameter HF will be equal to the area of the ellipse ABCD. *Q. E. D.*

SCHOLIUM.

From this and the two preceeding Propositions, a method is derived of finding the area of an ellipse. There are two ways: 1st, Say, as one is to the lesser diameter, so is the greater diameter to a fourth number, (which is found by the rule of three. Then again say, as 14 to 11, so is the 4th number found to the area sought. But the second way is shorter. Multiply the lesser diameter into the greater, and the product by 11; then divide the whole product by 14, and the quotient will be the area sought of the ellipse. For example, let the greater diameter be 10, and the lesser 7, by multiplying 10 by 7, the product is 70; and multiplying that by 11, it is 770; and dividing 770 by 14, the quotient will be 55, which is the area of the ellipse sought.

"The area of the ellipse will be found

"more

" more accurately, by multiplying the pro-
" duct of the two diameters by 7854."

We shall add no more about other plain surfaces, whether rectilinear or curvilinear, which seldom occur in practice; but shall subjoin some propositions about measuring the surfaces of solids.

PROP. IX. PROB.

To measure the surface of any prism.

BY the 14th definition of the 11th *Eucl.* a prism is contained by planes, of which two opposite sides (commonly called the bases) are plain rectilineal figures; which are either regular and ordinate, and measured by Prop. 5. of this part; or however irregular, and then they are measured by the 3d Prop. of this book. The other sides are parallelograms, which are measured by the 1st Prop. of this second part; and the whole superficies of the prism consists of the sum of those taken altogether.

PROP.

PROP. X. PROB.

To measure the superficies of any pyramid.

SINCE its basis is a rectilineal figure, and the rest of the planes terminating in the top of the pyramid are triangles; these measured separately, and added together, give the surface of the pyramid required.

PROP. XI. PROB.

To measure the superficies of any regular body.

THESE bodies are called regular, which are bounded by equilateral and equiangular figures. The superficies of the tetraedron consists of four equal and equiangular triangles; the superficies of the hexaedron, or cube, of six equal squares; an octædron, of eight equal equilateral triangles; a dodecaedron, of twelve equal and ordinate pentagons; and the superficies of an icosiædron, of twenty equal and equilateral triangles. Therefore it will be easy to measure these surfaces from what has been already shown.

In the same manner we may measure the superficies of a solid contained by any planes.

PROP.

PROP. XII.

PROB. FIG. 8.

To measure the superficies of a cylinder.

BECAUSE a cylinder differs very little from a prism whose opposite planes (or bases) are ordinate figures of an infinite number of sides, it appears that the superficies of a cylinder, without the bases, is equal to an infinite number of parallelograms; the common altitude of all which is, with the height of the cylinder, and the bases of them all differ very little from the periphery of the circle which is the base of the cylinder. Therefore this periphery multiplied into the common height, gives the superficies of the cylinder, excluding the bases; which are to be measured separately by the help of the 6th Prop. of this part.

This Proposition concerning the measure of the surface of the cylinder (excluding its basis) is evident from this, That when it is conceived to be spread out, it becomes a parallelogram, whose base is the periphery of the circle of the base of the cylinder stretched into a right

right line, and whose height is the same with the height of the cylinder.

PROP. XIII.

PROB. FIG. 9.

To measure the surface of a right cone.

THE surface of a right cone is very little different from the surface of a right pyramid, having an ordinate polygon for its base of an infinite number of sides; the surface of which (excluding the base) is equal to the sum of the triangles. The sum of the bases of these triangles is equal to the periphery of the circle of the base, and the common height of the triangles is the side of the cone AB: wherefore the sum of these triangles is equal to the product of the sum of the bases (*i. e.* the periphery of the base of the cone) multiplied into the half of the common height, or it is equal to the product of the periphery of the base.

If the area of the base is likewise wanted, it is to be found separately by the 6th Prop. of this part. If the surface of a cone is supposed

posed to be spread out on a plane, it will become a sector of a circle, whose radius is the side of the cone; and the arch terminating the sector is made from the periphery of the base. Whence, by Corol. 6. Prop. of this, its dimension may be found.

COROLLARY.

Hence it will be easy to measure the surface of a *frustum* of a cone cut by a plane parallel to the base. As to what relates to the measuring of the surface of the scalenous cone, because it is not very useful in practice, we shall not describe the method; which would carry us beyond the limits of this treatise.

PROP. XIV.

PROB. FIG. 10.

To measure the surface of a given sphere.

LET there be a sphere, whose center is A, and let the area of its convex surface be required. *Archimedes* demonstrates (37. Prop. 1. book of the sphere and cylinder) that its surface is equal to the area of four great cir-

cles of the ſphere; that is, let the area of the great circle be multiplied by 4, and the product will give the area of the ſphere; or, by 20th 6, and 2d 12. of *Eucl.* the area of the ſphere given is equal to the area of a circle whoſe radius is the right line BC, the diameter of the ſphere. Therefore having meaſured (by 6th Prop. of this part) the circle deſcribed with the radius BC, this will give the ſurface of the ſphere.

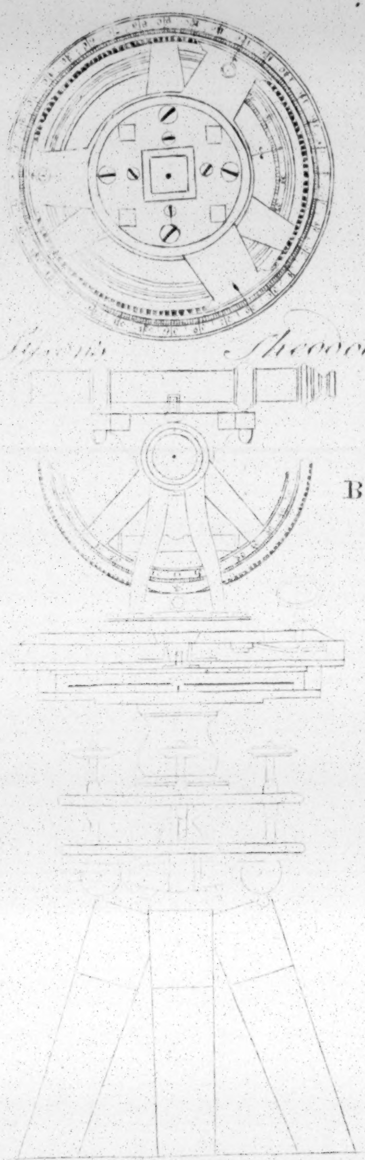
PROP. XV.

PROB. FIG. 10.

To meaſure the ſurface of a ſegment of a ſphere.

LET there be a ſegment cut off by the plane ED. *Archimedes* demonſtrates (49. and 50. 1. *de ſphæra*) that the ſurface of this ſegment, excluding the circular baſe, is equal to the area of a circle whoſe radius is the right line BE drawn from the vertex B of the ſegment to the periphery of the circle DE. Therefore, by the 6th Prop. of this part, it is eaſily meaſured.

COROL-



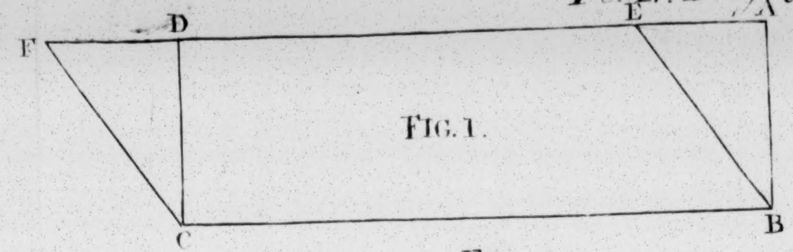


FIG. 2.

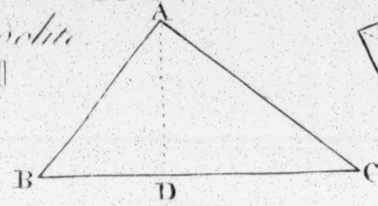


FIG. 3.

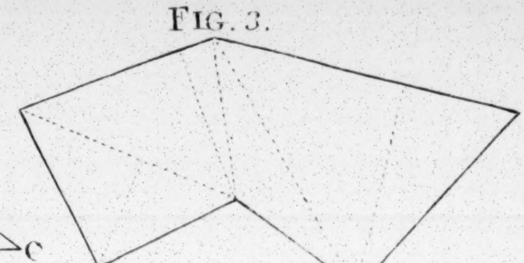


FIG. 4.

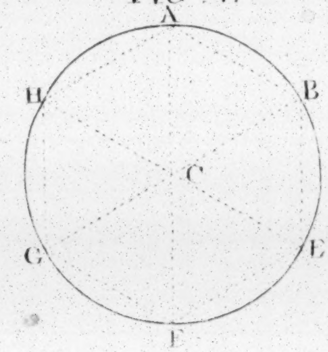
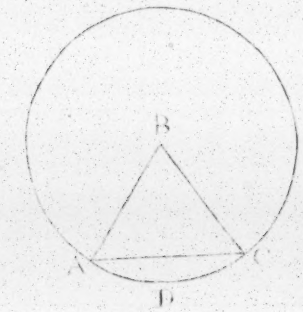


FIG. 5.



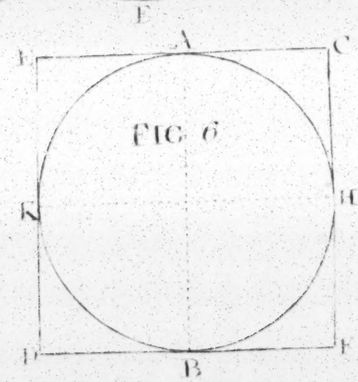


FIG. 7

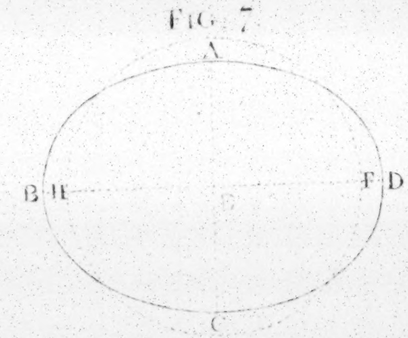


FIG. 8.

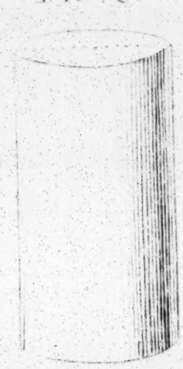


FIG. 9.

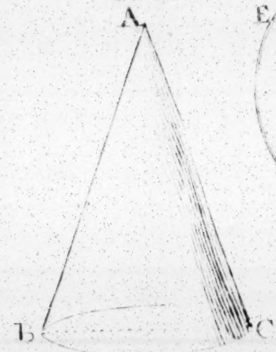


FIG. 10

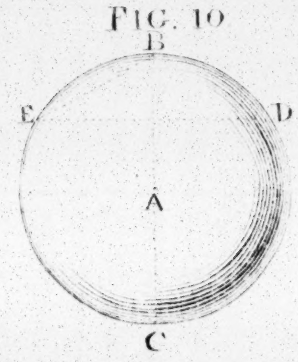
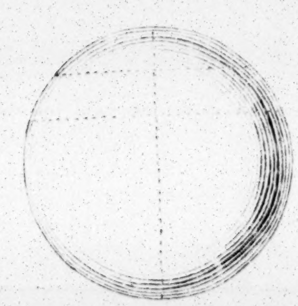


FIG. 11.



COROLLARY 1.

Hence that part of the surface of a sphere that lieth between two parallel planes is easily measured, by subtracting the surface of the lesser segment from the surface of the greater segment.

COROLLARY 2.

Hence likewise it follows, that the surface of a cylinder described about a sphere (excluding the basis) is equal to the surface of the sphere, and the parts of the one to the parts of the other, intercepted between planes parallel to the basis of the cylinder.

PART III.

Of solid figures and their mensuration.

AS in the preceeding parts we took an inch for the smallest measure in length, and an inch square for the smallest superficial measure; so now, in treating of the mensuration of solids, we take a cubical inch for the smallest solid measure. Of these 109 make a *Scots* pint; other liquid measures depend on this, as is generally known.

In

In dry meaſures, the firlot, by ſtatute, contains $19\frac{1}{2}$ pints; and on this depend the other dry meaſures: therefore, if the content of any ſolid be given in cubical inches, it will be eaſy to reduce the ſame to the common liquid or dry meaſures, and converſely to reduce theſe to ſolid inches. The liquid and dry meaſures in uſe among other nations, are known from their writers.

" As to the *Engliſh* liquid meaſures, by " act of Parliament 1706, any round veſſel, " commonly called a cylinder, having an " even bottom, being ſeven inches in diame- " ter throughout, and ſix inches deep from " the top of the inſide to the bottom, (which " veſſel will be found by computation to con- " tain $230\frac{907}{1000}$ cubical inches); or any veſſel " containing 231 cubical inches, and no more, " is deemed to be a lawful wine-gallon. " An *Engliſh* pint therefore contains $28\frac{7}{8}$ cu- " bical inches; two pints make a quart; four " quarts a gallon; 18 gallons a roundlet; " three roundlets and an half, or 63 gallons, " make a hogſhead; the half of a hogſhead " is a barrel; one hogſhead and a third, or " 84 gallons, make a puncheon; one pun-

" cheon

"cheon and a half, or two hogſheads, or "126 gallons, make a pipe or butt; the "third part of a pipe, or 42 gallons, make a "tierce; two pipes, or three puncheons, or "four hogſheads, make a ton of wine. "Though the *Engliſh* wine-gallon is now fix-"ed at 231 cubical inches, the ſtandard kept "in *Guildhall* being meaſured, before many "perſons of diſtinction, *May* 25. 1688, it was "found to contain only 224 ſuch inches.

"In the *Engliſh* beer-meaſure, a gallon "contains 282 cubical inches; conſequently "$35\frac{1}{4}$ cubical inches make a pint, two pints "make a quart, four quarts make a gallon, "nine gallons a firkin, four firkins a barrel. "In ale, eight gallons make a firkin, and 32 "gallons make a barrel. By an act of the "firſt of *William* and *Mary*, 34 gallons is "the barrel, both for beer and ale, in all "places, except within the weekly bills of "mortality.

"In *Scotland*, it is known that four gills "make a mutchkin, two mutchkins make a "chopin, a pint is two chopins, a quart is "two pints, and a gallon is four quarts or "eight pints. The accounts of the cubical "inches

" inches contained in the *Scots* pint vary " conſiderably from each other. According " ing to our Author, it contains 109 cubical inches. But the ſtandard-jugs kept " by the Dean of Guild of *Edinburgh* (one " of which has the year 1555, with the " arms of *Scotland*, and of the town of *Edinburgh* marked upon it) having been carefully meaſured ſeveral times, and by different perſons, the *Scots* pint, according " to thoſe ſtandards, was found to contain " about $103\frac{4}{10}$ cubic inches. The Pewterers jugs (by which the veſſels in common uſe are made) are ſaid to contain " ſometimes betwixt 105 and 106 cubic " inches. A caſk that was meaſured by the " Brewers of *Edinburgh*, before the Commiſſioners of Exciſe in 1707, was found " to contain $46\frac{7}{8}$ *Scots* pints; the ſame veſſel " contained $18\frac{1}{16}$ *Engliſh* ale-gallons. Suppoſing this menſurating to be juſt, the " *Scots* pint will be to the *Engliſh* ale-gallon " as 289 to 750; and if the *Engliſh* alegallon be ſuppoſed to contain 282 cubical inches, the *Scots* pint will contain " 108.664 cubical inches. But it is ſuſpected,

" on

" on several grounds, that this experiment was " not made with sufficient care and exactness.

" The Commissioners appointed by autho- " rity of Parliament to settle the measures " and weights, in their act of *February* 19. " 1618, relate, That having caused fill the " *Linlithgow* firlot with water, they found " that it contained $21\frac{1}{4}$ pints of the just " *Stirling* jug and measure. They likewise " ordain that this shall be the just and only " firlot, and add, *That the wideness and bread-* " *ness of the which firlot, under and above,* " *even over within the buirds, shall contain* " *nineteen inches and the sixth part of an inch,* " *and the deepness seven inches and a third* " *part of an inch.* According to this act " (supposing their experiment and computa- " tion to have been accurate) the pint con- " tained only 99.56 cubical inches; for the " content of such a vessel as is described in " the act is 2115.85, and this divided by " $21\frac{1}{4}$, gives 99.56. But, by the weight " of water said to fill this firlot in the same " act, the measure of the pint agrees nearly " with the *Edinburgh* standard above men- " tioned.

" As

"As for the *English* measures of corn, the
"*Winchester* gallon contains $272\frac{1}{4}$ cubical inch-
"es, two gallons make a peck, four pecks, or
"eight gallons (that is 2178 cubical inches)
"make a bushel, and a quarter is eight bushels.

"Our Author says, that $19\frac{1}{2}$ *Scots* pints
"make a firlot. But this does not appear
"to be agreeable to the statute above men-
"tioned, nor to the standard-jugs. It may
"be conjectured that the proportion assigned
"by him has been deduced from some expe-
"riment of how many pints, according to
"common use, were contained in the fir-
"lot. For if we suppose those pints to have
"been each of 108.664 cubical inches, ac-
"cording to the experiment made in the
"1707, before the Commissioners of Ex-
"cise described above; then $19\frac{1}{2}$ such pints
"will amount to 2118.94 cubical inches,
"which agrees nearly with 2115.85, the
"measure of the firlot by the statute above
"mentioned. But it is probable, that in
"this he followed the act 1587, where it is
"ordained, That the wheat-firlot shall con-
"tain 19 pints and two joucattes. A wheat-
"firlot marked with the *Linlithgow* stamps
"being

" being measured, was found to contain a-
" bout 2211 cubical inches. By the statute
" of 1618, the barley-firlot was to contain
" 31 pints of the just *Stirling* jug.

" A *Paris* pint is 48 cubical *Paris* inches,
" and is nearly equal to an *English* wine
" quart. The *Boissean* contains 644.68099
" *Paris* cubical inches, or 780.36 *English*
" cubical inches.

" The *Roman Amphora* was a cubical *Ro-*
" *man* foot, the *Congius* was the 8th part
" of the *Amphora*, the *Sextarius* was one
" sixth of the *Congius*. They divided the
" *Sextarius* like the *As* or *Libra*. Of dry
" measures, the *Medimnus* was equal to two
" *Amphoras*, that is about $1\frac{1}{2}\frac{1}{5}$ *English* legal
" bushels; and the *Medius* was the third part
" of the *Amphora*."

PROP. I. PROB.

To find the solid content of a given prism.

BY the 2d Prop. of the 2d part of this, let the area of the base of the prism be measured, and be multiplied by the height

of the prism, the product will give the solid content of the prism.

PROP. II. Prob.

To find the solid content of a given pyramid.

THE area of the base being found (by the 3d Prop. of the 2d part) let it be multiplied by the third part of the height of the pyramid, or the third part of the base by the height, the product will give the solid content, by 7th 12. *Eucl.*

COROLLARY.

If the solid content of a *frustum* of a pyramid is required, first let the solid content of the entire pyramid be found; from which subtract the solid content of the part that is wanting, and the solid content of the broken pyramid will remain.

PROP. III. Prob.

To find the content of a given cylinder.

THE area of the base being found (by Prop. 6. of the 2d part) if it be a circle,

and by Prop. 8. if it be an ellipſe (for in both caſes it is a cylinder) multiply it by the height of the cylinder, and the ſolid content of the cylinder will be produced.

COROLLARY. Fig. I.

And in this manner may be meaſured the ſolid content of veſſels and caſks not much different from a cylinder as ABCD. If towards the middle EF it be ſomewhat groſſer, the area of the circle of the baſe being found (by 6th Prop. of the 2d part) and added to the area of the middle circle EF, and the half of their ſum (that is an arithmetical mean between the area of the baſe, and the area of the middle circle) taken for the baſe of the veſſel, and multiplied into its height, the ſolid content of the given veſſel will be produced.

Note, That the length of the veſſel, as well as the diameters of the baſe, and of the circle EF, ought to be taken within the ſtaves; for it is the ſolid content within the ſtaves that is ſought.

PROP.

PROP. IV. PROB.

To find the solid content of a given cone.

LET the area of the base (found by Prop. 6. 2d part) be multiplied into $\frac{1}{3}$ of the height, the product will give the solid content of the cone; (for by 10th 12. *Eucl.* a cone is the third part of a cylinder that has the same base and height.

PROP. V.

PROB. FIG. 2. and 3.

To find the solid content of a frustum *of a cone cut by a plane parallel to the plane of the base.*

FIRST let the height of the entire cone be found, and thence (by the preceeding Prop.) its solid content; from which subtract the solid content of the cone cut off at the top, there will remain the solid content of the *frustum* of the cone.

How the content of the entire cone may be found, appears thus: let ABCD be the *frustum* of the cone (either right or scalenous,

nous, as in the figures 2. and 3.) ; let the cone ECD be ſuppoſed to be compleated ; let AG be drawn parallel to DE, and let AH and EF be perpendicular on CD; it will be (by 2d 6. *Eucl.*) as CG: CA : : CD: CE ; but (by the 4th Prop. of the ſame book) as CA: AH : : CE: EF ; conſequently (by 22d 5. *Eucl.*) as CG: AH : : CD: EF ; that is, as the exceſs of the diameter of the leſſer baſe, is to the height of the *fruſtum*, ſo is the diameter of the greater baſe to the height of the entire cone.

COROLLARY. Fig. 4.

Some caſks whoſe ſtaves are remarkably bended about the middle, and ſtreight to wards the ends, may be taken for two portions of cones, without any conſiderable error. Thus ABEF is a *fruſtum* of a right cone, to whoſe baſe EF, on the other ſide, there is another ſimilar *fruſtum* of a cone joined EDCF. The vertices of theſe cones, if they be ſuppoſed to be compleated, will be found at G and H. Whence, by the preceeding Prop. the ſolid content of ſuch veſſels may be found.

PROP.

PROP. VI.
THEOR. FIG. 5.

A Cylinder circumscribed about a sphere, that is, having its base equal to a great circle of the sphere, and its height equal to the diameter of the sphere, is to the sphere as 3 to 2.

Let ABEC be the quadrant of a circle, and ABDC the circumscribed square; and likewise the triangle ADC; by the revolution of the figure about the right line AC as axis, a hemisphere will be generated by the quadrant, a cylinder of the same base and height by the square, and a cone by the triangle. Let these three be cut any how by the plane HF parallel to the base AB, the section in the cylinder will be a circle, whose radius is FH; in the hemisphere a circle of the radius EF; and in the cone, a circle of the radius GF.

By the 47th 1. *Eucl.* EAq, or $HFq=EFq$ and FAq taken together, (but $AFq=FGq$, because $AC=CD$); therefore the circle of the radius HF, is equal to a circle of the radius EF together with a circle of the

radius

radius GF ; and ſince this is true every where, all the circles together deſcribed by the reſpective *radii* HF (that is the cylinder) are equal to all the circles deſcribed by the reſpective *radii* EF and FG (that is to the hemiſphere and the cone taken together); but, by 10th 12. *Eucl.* the cone generated by the triangle DAC is one third part of the cylinder generated by the ſquare BC. Whence it follows, that the hemiſphere generated by the rotation of the quadrant ABEC, is equal to the remaining two third parts of the cylinder, and that the whole ſphere is $\frac{2}{3}$ of the double cylinder circumſcribed about it.

This is that celebrated 39th Prop. 1. book of *Archimedes* of the ſphere and cylinder ; in which he determines the proportion of the cylinder to the ſphere inſcribed to be that of 3 to 2.

COROLLARY.

Hence it follows, that the ſphere is equal to a cone, whoſe height is equal to the ſemidiameter of the ſphere, having for its baſe a a circle equal to the ſuperficies of the ſphere, or

to

to four great circles of the ſphere, or to a circle whoſe radius is equal to the diameter of the ſphere, by 14th Prop. 2d part of this. And indeed a ſphere differs very little from the ſum of an infinite number of cones, that have their baſes in the ſurface of the ſphere, and their common vertex in the centre of the ſphere: ſo that the ſuperficies of the ſphere, (of whoſe dimenſion ſee 14th Prop. 2d part of this) multiplied into the third part of the ſemidiameter, gives the ſolid content of the ſphere.

PROP. VII.

PROB. FIG. 6.

To find the ſolid content of a ſector of the ſphere.

A Spherical ſector ABC (as appears by the Corol. of the preceeding Prop.) is very little different from an infinite number of cones, having their baſes in the ſuperficies of the ſphere BEC, and their common vertex in the centre. Wherefore the ſpherical ſuperficies BEC being found (by 15. Prop. 2d part),

part), and multiplied into the third part of AB the radius of the ſphere, the product will give the ſolid content of the ſector ABC.

COROLLARY.

It is evident how to find the ſolidity of a ſpherical ſegment leſs than a hemiſphere, by ſubtracting the cone ABC from the ſector already found. But if the ſpherical ſegment be greater than a hemiſphere, the cone correſponding muſt be added to the ſector, to make the ſegment.

PROP. VIII.

PROB. FIG. 7.

To find the ſolidity of the ſpheroid, and of its ſegments cut by planes perpendicular to the axis.

IN the 2d Prop. of this part, it is ſhown that every where EH : EG : : CF : CD ; but circles are as the ſquares deſcribed upon their rays, that is, the circle of the radius EH, is to the circle of the radius EG, as

CFq to CDq. And ſince it is ſo every where, all the circles deſcribed with the reſpective rays EH (that is the ſpheroid made by the rotation of the ſemiellipſis AFB around the axis AB) will be to all the circles deſcribed by the reſpective *radii* EG (that is the ſphere deſcribed by the rotation of the ſemicircle ADB on the axis AB) as FCq to CDq; that is, as the ſpheroid to the ſphere on the ſame axis, ſo is the ſquare of the other axis of the generating ellipſe to the ſquare of the axis of the ſphere.

And this holds, whether the ſpheroid be found by a revolution around the greater or leſſer axis.

COROLLARY 1.

Hence it appears, that the half of the ſpheroid, formed by the rotation of the ſpace AHFC around the axis AC, is double of the cone generated by the triangle AFC about the ſame axis; which is the 32d Prop. of *Archimedes*, of conoids and ſpheroids.

COROLLARY 2.

Hence likewiſe is evident the meaſure of ſegments of the ſpheroid cut by planes perpendicular

pendicular to the axis. For the ſegment of the ſpheroid made by the rotation of the ſpace ANHE, round the axis AE, is to the ſegment of the ſphere having the ſame axis AC, and made by the rotation of the ſegment of the circle AMGE, as CFq to CDq.

But if the meaſure of this ſolid be wanted with leſs labour, by the 34th Prop. of *Archimedes*, of conoids and ſpheroids, it will be as BE to AC + EB, ſo is the cone generated by the rotation of the triangle AHE round the axis AE, to the ſegment of the ſphere made by the rotation of the ſpace ANHE round the ſame axis AE; which could eaſily be demonſtrated, (was this a proper place for it), by the method of indiviſibles.

COROLLARY 3.

Hence it is eaſy to find the ſolid content of the ſegment of a ſphere or ſpheroid intercepted between two parallel planes, perpendicular to the axis. This agrees as well to the oblate as to the oblong ſpheroid; as is obvious.

COROL-

COROLLARY 4. Fig. 8.

If a caſk is to be valued as the middle piece of an oblong ſpheroid, cut by the two planes DC and FG, at right angles to the axis. Firſt let the ſolid content of the half ſpheroid ABCED be meaſured by the preceeding Prop.; from which let the ſolidity of the ſegment DEC be ſubtracted, and there will remain the ſegment ABCD; and this doubled, will give the capacity of the caſk required.

The following method is generally made uſe of for finding the ſolid content of ſuch veſſels. The double area of the greateſt circle, that is of that which is deſcribed by the diameter AB at the middle of the caſk, is added to the area of the circle at the end, that is of the circle DC or FG (for they are uſually equal), and the third part of this ſum is taken for a mean baſe of the caſk; which therefore multiplied into the length of the caſk OP, gives the content of the veſſel required.

Sometimes veſſels have other figures, different from thoſe we have mentioned; the eaſy methods of meaſuring which may be learned

learned from those who practise this art. What hath already been delivered, is sufficient for our purpose.

PROP. IX.

PROB. FIG. 9. and 10.

To find how much is contained in a vessel that is in part empty, whose axis is parallel to the horizon.

LET AGBH be the great circle in the middle of the cask, whose segment GBH is filled with liquor, the segment GAH being empty; the segment GBH is known, if the depth EB be known, and EH a mean proportional between the segments of the diameter AE and EB; which are found by a rod or ruler put into the vessel at the orifice. Let the basis of the cask, at a medium, be found, which suppose to be the circle CKDL; and let the segment KCL be similar to the segment GAH (which is either found by the rule of three, because as the circle AGBH is to the circle CKDL, so is the segment GAH to the segment KCL;

KCL; or is found from the tables of segments made by authors); and the product of this segment multiplied by the length of the cask, will give the liquid content remaining in the cask.

PROP. X. PROB.

To find the solid content of a regular and ordinate body.

A Tetraedron being a pyramid, the solid content is found by the 2d Prop. of this part. The Hexaedron, or cube, being a kind of prism, it is measured be the 1st Prop. of this part. An Octaedron consists of two pyramids of the same square base and of equal heights; consequently its measure is found from the second Prop. of this part. A Dodecaedron consists of twelve pyramids having equal equilateral and equiangular pentagonal bases; and so one of these being measured (by 2d Prop. of this) and multiplied by 12, the product will be equal to the solid content of the Dodecaedron. The Icosiaedron consists of 20 equal pyramids having triangular bases; the

solid

ſolid content of one of which being found (by the 2d Prop. of this) and multiplied by 20, gives the whole ſolid. The baſes and heights of theſe pyramids, if you want to proceed more exactly, may be found by Trigonometry.

PROP. XI. PROB.

To find the ſolid content of a body, however irregular.

LET the given body be immerſed into a veſſel of water, having the figure of a parallelopipedon or priſm, and let it be noted how much the water is raiſed upon the immerſion of the body. For it is plain that the ſpace which the water fills, after the immerſion of the body, exceeds the ſpace filled before its immerſion, by a ſpace equal to the ſolid content of the body however irregular. But when this exceſs is of the figure of a parallelopipedon or priſm, it is eaſily meaſured by the firſt Prop. of this part, to wit, by multiplying the area of the baſe, or mouth of the veſſel, into the difference of the elevations of the water before

fore and after immersion. Whence is found the solid content of the body given. *Q. E. I.*

In the same way the solid content of a part of a body may be found, by immersing that part only in water.

There is no necessity to insist here on diminishing or enlarging solid bodies in a given proportion. It will be easy to deduce these things from the 11th and 12th books of *Euclid*.

" The following rules are subjoined for the " ready computation of the contents of vessels, and of any solids in the measures in " use in *Great Britain*.

" I. To find the content of a cylindric vessel in *English* wine-gallons, the diameter of " the base and altitude of the vessel being " given in inches and decimals of an inch.

" Square the number of inches in the " diameter of the vessel; multiply this square " by the number of inches in the height: " then multiply the product by the decimal fraction .0034; and this last product " shall give the content in wine-gallons and " decimals of such a gallon. To express " the rule arithmetically; let D represent

" the

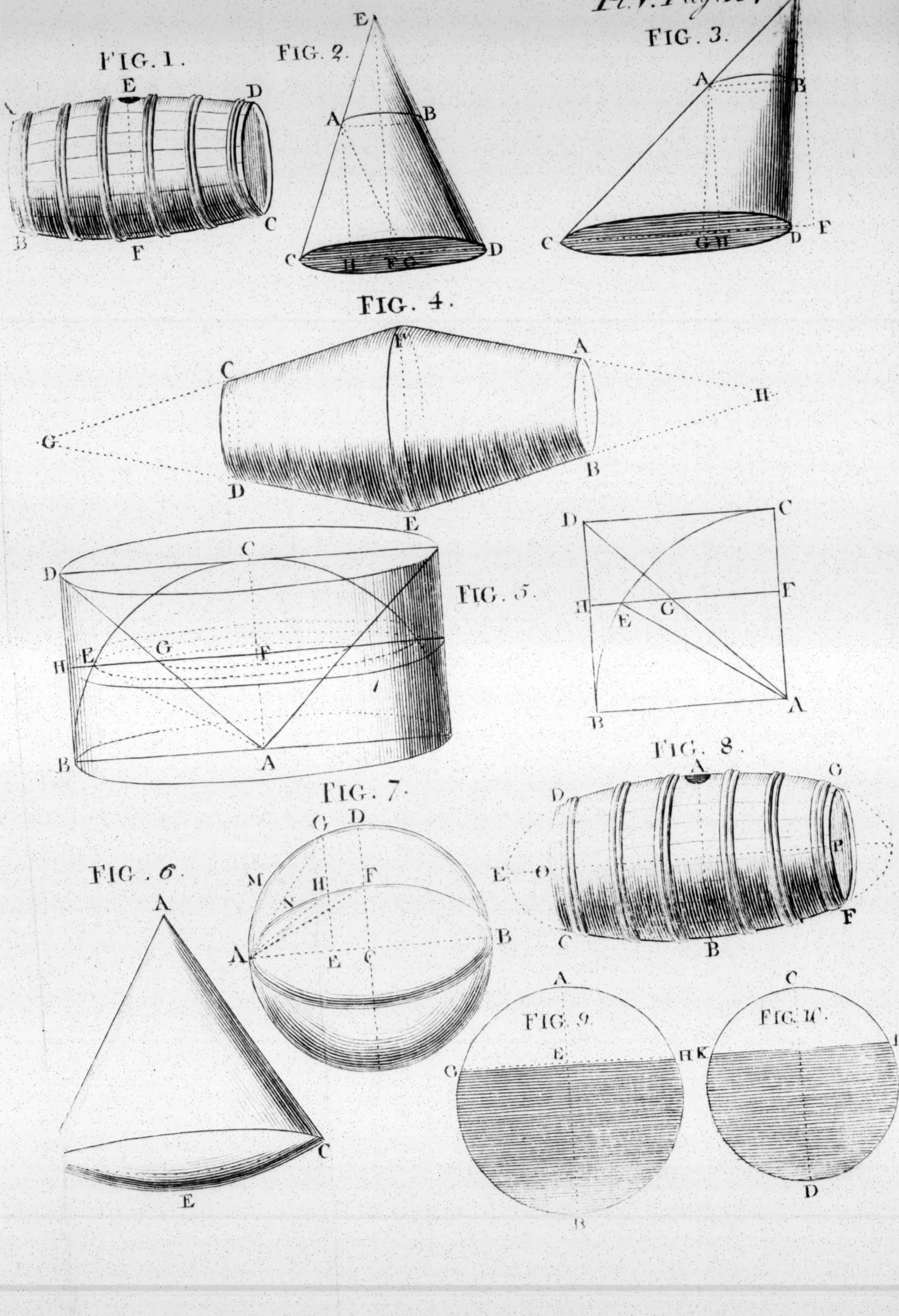


FIG. 1.
FIG. 2.
FIG. 3.
FIG. 4.
FIG. 5.
FIG. 6.
FIG. 7.
FIG. 8.
FIG. 9.
FIG. 10.

"the number of inches and decimals of "an inch in the diameter of the veſſel, "and H the inches and decimals of an "inch in the height of the veſſel; then the "content in wine-gallons ſhall be DDH "$\times \frac{34}{10000}$, or DDH $\times$ .0034. *Ex.* Let the "diameter D=51.2 inches, the height H "=62.3 inches, then the content ſhall be "51.2 $\times$ 51.2 $\times$ 62.3 $\times$ .0034=555.27,342 "wine-gallons. This rule follows from "Prop. 7. of the ſecond part, and Prop. "3. of the third part; for, by the former, "the area of the baſe of the veſſel is in "ſquare inches DD $\times$ .7854; and by the "latter, the content of the veſſel in ſolid in-"ches is DDH $\times$ 7854; which divided by "231 (the number of cubical inches in a "wine-gallon) gives DDH $\times$ .0034, the con-"tent in wine-gallons. But tho' the char-"ges in the Exciſe are made (by ſtatute) on "the ſuppoſition that the wine-gallon con-"tains 231 cubical inches, yet it is ſaid, that "in ſale 224 cubical inches, the content of "the ſtandard meaſured in *Guildhall* (as was "mentioned above) are allowed to be a "wine-gallon.

" II. Supposing the *English* ale-gallon
" to contain 282 cubical inches, the content
" of a cylindric vessel is computed in such
" gallons, by multiplying the square of the
" diameter of a vessel by its height as for-
" merly, and their product by the decimal
" fraction .0,027,851. That is, the solid con-
" tent in ale-gallons is DDH×.0,027,851.

" III. Supposing the *Scots* pint to con-
" tain about 103.4 cubical inches (which is
" the measure given by the standards at *E-*
" *dinburgh*, according to experiments men-
" tioned above) the content of a cylindric ves-
" sel is computed in *Scots* pints, by multiply-
" ing the square of the diameter of the vessel
" by its height, and the product of these by
" the decimal fraction .0076. Or the con-
" tent of such a vessel in *Scots* pints is DDH
" ×.0076.

" IV. Supposing the *Winchester* bushel to
" contain 2187 cubical inches, the content
" of a cylindric vessel is computed in those
" bushels by multiplying the square of the
" diameter of the vessel by the height, and
" the product by the decimal fraction
" .0,003,606. But the standard bushel ha-
" ving been measured by Mr *Everard* and
" others

" others in 1696, it was found to contain " only 2145.6 solid inches, and therefore " it was enacted in the act for laying a duty " upon malt, *That every round bushel, with " a plain and even bottom, being* $18\frac{1}{2}$ *inches dia- " meter throughout, and* 8 *inches deep, should " be esteemed a legal* Winchester *bushel.* Ac- " cording to this act (ratified in the first year " of Queen *Anne*) the legal *Winchester* bushel " contains only 2150.42 solid inches. And " the content of a cylindric vessel is computed " in such bushels, by multiplying the square " of the diameter by the height, and their " product by the decimal fraction .0,003,652. " Or the content of the vessel in those " bushels is DDH×.0,003,625.

" V. Supposing the *Scots* wheat-firlot to " contain $21\frac{1}{2}$ *Scots* pints, (as is appointed " by the statute 1618), and the pint to be " conform to the *Edinburgh* standards above " mentioned, the content of a cylindric ves- " sel in such firlots is computed by multiply- " ing the square of the diameter by the " height, and their product by the decimal " fraction .00,358. This firlot, in 1426, is " appointed to contain 17 pints; in 1457,

" it

" it was appointed to contain 18 pints; in " 1587, it is $19\frac{1}{4}$ pints; in 1628, it is $21\frac{1}{4}$ " pints: and though this laſt ſtatute appears " to have been found on wrong computa- " tions in ſeveral reſpects, yet this part of the " act that relates to the number of pints in the " firlot ſeems to be the leaſt exceptionable; " and therefore we ſuppoſe the firlot to con- " tain $21\frac{1}{4}$ pints of the *Edinburgh* ſtandard, " or about 2197 cubical inches; which a " little exceeds the *Wincheſter* buſhel, from " which it may have been originally copied.

" VI. Suppoſing the bear-firlot to contain " 31 *Scots* pints (acording to the ſtatute " 1618) and the pint conform to the *Edin-* " *burgh* ſtandards, the content of a cylindric " veſſel in ſuch firlots is found by multiply- " ingthe ſquare of the diameter bythe height, " and this product by .000,245.

" When the ſection of the veſſel is not a cir- " cle, but an ellipſis, the product of the great- " eſt diameter by the leaſt is to be ſubſtituted " in thoſe rules for the ſquare of the diameter.

" VII. To compute the content of a veſ- " ſel that may be conſidered as a *fruſtum* of " a cone in any of thoſe meaſures.

" Let

" Let A represent the number of inches " in the diameter of the greater base, B " the number of inches in the diameter of " the lesser base. Compute the square of " A, the product of A multiplied by B, and " the square of B, and collect these into " a sum. Then find the third part of this " sum, and substitute it in the preceeding " rules in the place of the square of the " diameter; and proceed in all other re- " spects as before. Thus, for exam- " ple, the content in wine-gallons is " $\overline{AA + AB + BB} \times \frac{1}{3} \times H \times .0034$.

" Or to the square of half the sum of the " diameters A and B, add one third part of " the square of half their difference; and " substitute this sum in the preceeding rules " for the square of the diameter of the ves- " sel; for the square of $\frac{1}{2}$ A + $\frac{1}{2}$ B added to " $\frac{1}{3}$ of the square of $\frac{1}{2}$ A —— $\frac{1}{2}$ B, gives $\frac{1}{3}$ " AA + $\frac{1}{3}$ AB + $\frac{1}{3}$ BB.

" VIII. When a vessel is a *frustum* of " a parabolic conoid, measure the diame- " ter of the section at the middle of the " height of the *frustum*; and the content " will be precisely the same as of a cylin- " der

"der of this diameter, of the ſame height
"with the veſſel.

"IX. When a veſſel is a *fruſtum* of a "ſphere, if you meaſure the diameter of the "ſection at the middle of the height of the "*fruſtum*, then compute the content of a "cylinder of this diameter of the ſame "height with the veſſel, and from this ſub-"tract $\frac{1}{3}$ of the content of a cylinder of the "ſame height on a baſe whoſe diameter is "equal to its height; the remainder will give "the content of the veſſel. That is, if D "repreſent the diameter of the middle ſection, "and H the height of the *fruſtum*, you are "to ſubſtitute $DD - \frac{1}{3}HH$ for the ſquare "of the diameter of the cylindric veſſel in "the firſt ſix rules.

"X. When the veſſel is a *fruſtum* of a "ſpheroid, if the baſes are equal, the con-"tent is readily found by the rule in *p.* 97. "In other caſes, let the axis of the ſolid "be to the conjugate axis, as n to 1; let D "be the diameter of the middle ſection of "the *fruſtum*, H the height or length of "the *fruſtum*; and ſubſtitute in the firſt "ſix rules $DD - \frac{HH}{3nn}$ for the ſquare of

"the

" the ſquare of the diameter of the veſ-
" ſel.

" XI. When the veſſel is an hyperbolic
" conoid, let the axis of the ſolid be to the
" conjugate axis, as n to 1, D the diameter
" of the ſection at the middle of the *fruſtum*,
" H, the height or length: compute $DD + \frac{1}{3nn} \times HH$, and ſubſtitute this ſum for the
" ſquare of the diameter of the cylindric
" veſſel in the firſt ſix rules.

" XII. In general, it is uſual to meaſure
" any round veſſel, by diſtinguiſhing it into
" ſeveral *fruſtums*, and taking the diameter
" of the ſection at the middle of each *fru-*
" *ſtum*; thence to compute the content of
" each, as if it was a cylinder of that mean
" diameter; and to give their ſum as the
" content of the veſſel. From the total
" content computed in this manner, they ſub-
" tract ſucceſſively the numbers which ex-
" preſs the circular areas that correſpond
" to thoſe mean diameters, each as often as
" there are inches in the altitude of the *fru-*
" *ſtum* to which it belongs, beginning with
" the uppermoſt; and in this manner cal-
" culate a table for the veſſel, by which
" it

" it readily appears how much liquor is at " any time contained in it, by taking either " the dry or wet inches ; having regard to " the inclination or drip of the vessel when " it has any.

" This method of computing the con- " tent of a *frustum* from the diameter of " the section at the middle of its height, " is exact in that case only when it is a " portion of a parabolic conoid; but in " such vessels as are in common use, the " error is not considerable. When the " vessel is a portion of a cone or hyperbolic " conoid, the content by this method is " found less than the truth ; but when it " is a portion of a sphere or spheroid, the " content computed in this manner exceeds " the truth. The difference or error is al- " ways the same, in the different parts of " the same or of similar vessels when the " altitude of the *frustum* is given. And " when the altitudes are different, the error " is in the triplicate *ratio* of the altitude. " If exactness be required, the error in mea- " suring the *frustum* of a conical vessel, in " this manner, is $\frac{1}{4}$ of the content of a

" cone,

"cone, similar to the vessel, of an altitude equal to the height of the *frustum*. In a sphere, it is $\frac{1}{3}$ of a cylinder of a diameter and height equal to the *frustum*. In the spheroid and hyperbolic conoid, it is the same as in a cone generated by the right-angled triangle contained by the two semiaxes of the figure revolving about that side which is the semiaxis of the *frustum*. These are demonstrated in a treatise of fluxions by Mr *Colin M'Laurin*, *p.* 22. and 715. where those theorems are extended to *frustums* that are bounded by planes oblique to the axis in all the solids that are generated by any conic section revolving about either axis.

"In the usual method of computing a table for a vessel, by subducting from the whole content the number that expresses the uppermost area as often as there are inches in the uppermost *frustum*, and afterwards the numbers for the other areas successively; it is obvious that the contents assigned by the table, when a few of the uppermost inches are dry, are stated a little too high, if the vessel stands on its

" leſſer baſe, but too low when it ſtands on " its greater baſe; becauſe, when one inch " is dry, for example, it is not the area at " the middle of the uppermoſt *fruſtum*, but " rather the area at the middle of the upper- " moſt inch, that ought to be ſubducted from " the total content, in order to find the con- " tent in this caſe.

" XIII. To meaſure round timber, let " the mean circumference be found in feet " and decimals of a foot; ſquare it; multi- " ply this ſquare by the decimal .079,577, " and the product by the length. *Ex.* Let " the mean circumference of a tree be 10.3 " feet, and the length 24 feet. Then " $10.3 \times 10.3 \times .079{,}577 \times 24 = 202.615$, " is the number of cubical feet in the tree. " The foundation of this rule is, that when " the circumference of a circle is 1, the a- " rea is .0,795,774,715, and that the areas " of circles are as the ſquares of their cir- " cumferences.

" But the common way uſed by artifi- " cers for meaſuring round timber, differs " much from this rule. They call one fourth " part of the circumference the *girt*, which

" is

" is by them reckoned the ſide of a ſquare, " whoſe area is equal to the area of the " ſection of the tree; therefore they ſquare " the *girt*, and then multiply by the length " of the tree. According to their method, " the tree of the laſt example would be " computed at 159.13 cubical feet only.

" How ſquare timber is meaſured, will be " eaſily underſtood from the preceeding Pro- " poſitions. Fifty ſolid feet of hewn tim- " ber, and forty of rough timber, make a " load.

" XIV. To find the burden of a ſhip, " or the number of tons it will carry, the " following rule is commonly given. Multi- " ply the length of the keel taken within " board, by the breadth of the ſhip within " board, taken from the midſhip-beam from " plank to plank, and the product by the " depth of the hold, taken from the plank " below the keelſon to the under part of the " upper deck plank, and divide the product " by 94, the quotient is the content of the " tonnage required. This rule however can- " not be accurate; nor can one rule be ſup- " poſed to ſerve for the meaſuring exactly

" the

" the burden of ſhips of all ſorts. Of this " the reader will find more in the Memoirs " of the royal academy of ſciences at *Paris* " for the year 1721.

" Our Author having ſaid nothing of " weights, it may be of uſe to add briefly, " that the *Engliſh* Troy-pound contains 12 " ounces, the ounce 20 penny-weight, and " the penny-weight 24 grains; that the *A-* " *verdupois* pound contains 16 ounces, the " ounce 16 drams, and that 112 pounds is " uſually called the hundred weight. It is " commonly ſuppoſed that 14 pounds *Aver-* " *dupois* are equal to 17 pounds Troy. Ac- " cording to Mr *Everard*'s experiments, one " pound *Averdupois* is equal to 14 ounces 11 " penny-weight and 16 grains Troy, that is, " to 7000 grains; and an *Averdupois* ounce " is $437\frac{1}{2}$ grains. The *Scots* Troy-pound " (which by the ſtatute 1718, was to be the " ſame with the *French*) is commonly ſup- " poſed equal to $15\frac{3}{4}$ ounces *Engliſh* Troy, " or 7560 grains. By a mean of the ſtan- " dards kept by the Dean of Guild of *Edin-* " *burgh*, it is $7599\frac{3}{32}$ or 7600 grains. They " who have meaſured the weights which

" were

" were ſent from *London*, after the union of
" the Kingdoms, to be the ſtandards by
" which the weights in *Scotland* ſhould be
" made, have found the *Engliſh Averdupois*
" pound (from a medium of the ſeveral
" weights) to weigh 7000 grains, the ſame
" as Mr *Everard;* according to which, the
" *Scots*, *Paris*, or *Amſterdam* pound, will
" be to the pound *Averdupois*, as 38 to 35.
" The *Scots* Troy-ſtone contains 16 pounds,
" the pound two marks or 16 ounces, an
" ounce 16 drops, a drop 36 grains. Twenty
" *Scots* ounces make a Trone pound; but
" becauſe it is uſual to allow one to the ſcore,
" the Trone pound is commonly 21 ounces.
" Sir *John Skene* however makes the Trone
" ſtone to contain only $19\frac{1}{2}$ pounds."

F I N I S.

James Craufford
1766

James Craufford 1766
James Craufford 1766

1766

The sum of the angles in every triangle is = to two right angles [illegible]